M 7148

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CCSS – Reg./Supple./Imp.) Examination, November 2014
CORE COURSE IN MATHEMATICS
5B05 MAT : Vector Analysis

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks :
   a) Midpoint of the line segment joining points (x_1, y_1, z_1) and (x_2, y_2, z_2) is ________
   b) Vector equation of the line through $P_0 (x_0, y_0, z_0)$ and parallel to $\vec{v}$ is ________
   c) If $\vec{r}$ is the position vector of a particle moving along a smooth curve in space, then velocity vector at any time t is ________
   d) The curvature of a straight line is ________ **(Weightage 1)**

Answer **any six** from the following (weightage **1 each**) :

2. Find the angle between $\vec{a} = \hat{i} - 2\hat{j} - 2\hat{k}$ and $\vec{b} = 6\hat{i} + 3\hat{j} + 2\hat{k}$.
3. Find the Cartesian equation for the surface $z = r^2$ and identity the surface.
4. If $f(x, y) = y \sin xy$, find $\frac{\partial f}{\partial y}$.
5. Find $\frac{dy}{dx}$ if $x^2 + \sin y - 2y = 0$.
6. Define gradient of a scalar field.
7. Evaluate $\iint_R (1 - 6x^2y)\,dx\,dy$ where R is the region between $x = 0, x = 2, y = -1$ and $y = 1$.
8. Evaluate $\int_0^1 \int_0^{2-x} \int_0^{2-x-y} dz\, dy\, dx$.
9. Find the divergence of $\vec{F} = (x^2 - y)\,\hat{i} + (xy - y^2)\,\hat{j}$.
10. State Green's theorem in plane. **(Weightage 6×1=6)**

P.T.O.

Answer **any seven** from the following (weightage **2 each**) :

11. Find parametric equations for the line through (–3, 2, –3) and (1, –1, 4).
12. Find a vector perpendicular to the plane of P(1, –1, 0), Q (2, 1, –1) and R (–1, 1, 2).
13. Find the length of one turn of the helix $\vec{r}(t) = \cos t\,\hat{i} + \sin t\,\hat{j} + t\hat{k}$.
14. Show that the function $f(x, y) = \dfrac{2x^2y}{x^4 + y^2}$ has no limit as (x, y) approaches (0, 0).
15. Find the linearization of $f(x, y, z) = x^2 - xy + 3\sin z$ at the point (2, 1, 0).
16. Find the derivative of $f(x, y) = x^2 + xy$ at (1, 2) in the direction of the vector $\hat{i} + \hat{j}$.
17. Change the order of integration and hence evaluate $\int_0^2 \int_{x^2}^{2x} (4x + 2)\,dy\,dx$.
18. Find the polar moment of inertia about the origin of a thin plate of density $\delta(x, y) = 1$ bounded by the quarter circle $x^2 + y^2 = 1$ in the first quadrant.
19. Show that $\vec{F} = (y \sin z)\,\hat{i} + (x \sin z)\,\hat{j} + (xy \cos z)\,\hat{k}$ is conservative and find a potential for it.
20. Find a parametrization of the cylinder $x^2 + (y - 3)^2 = 9, 0 \le z \le 5$.

(Weightage 7×2=14)

Answer **any three** from the following (weightage **3 each**) :

21. Find the local extreme values of the function $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$.
22. Find the volume of the region D enclosed by the surfaces $z = x^2 + 3y^2$ and $z = 8 - x^2 - y^2$.
23. Evaluate

$$\int_0^3 \int_0^4 \int_{x=y/2}^{x=(y/2)+1} \left(\frac{2x - y}{2} + \frac{z}{3}\right) dx\,dy\,dz$$

by applying the transformation $u = \dfrac{2x - y}{2}, v = \dfrac{y}{2}, w = \dfrac{z}{3}$.

24. Find the circulation of the field $\vec{F} = (x^2 - y)\,\hat{i} + 4z\,\hat{j} + x^2\,\hat{k}$ around the curve C in which the plane z = 2 meets the cone $z = \sqrt{x^2 + y^2}$ counterclockwise as viewed from above.
25. Verify divergence theorem for $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$ over the sphere $x^2 + y^2 + z^2 = a^2$.

(Weightage 3×3=9)

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