**M 9811**

**Reg. No. :** ..............................

**Name :** ..................................

**V Semester B.Sc. Degree (CCSS – Reg./Supple./Imp.) Examination, November 2015**
**CORE COURSE IN MATHEMATICS**
**5 B05 MAT : Vector Analysis**

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks :
   a) Two non-zero vectors $\vec{a}$ and $\vec{b}$ are orthogonal if and only if ________
   b) Vector equation of the plane through $P_0$ ($x_0$, $y_0$, $z_0$) and normal to $\vec{n}$ is ________
   c) If $\vec{v}$ is the velocity vector of a particle moving along a smooth curve in space at time t, then acceleration vector is ________
   d) If k is the curvature then center of curvature $\rho$ is ________ **(Weightage 1)**

Answer **any six** from the following **(Weightage 1 each) :**

2. Find the area of the triangle with vertices (1, –1, 0), (2, 1, –1) and (–1, 1, 2).
3. Find the equation of the cylinder $x^2 + (y-3)^2 = 9$ in cylindrical coordinates.
4. If $f(x, y) = \dfrac{2y}{y + \cos x}$, find $f_x$.
5. Find $\dfrac{dw}{dt}$ at $t = \dfrac{\pi}{2}$ if w = xy, x = cos t, y = sin t.
6. State Euler's theorem on homogeneous functions.
7. Find the area of the region R bounded by y = x and $y = x^2$ in the first quadrant.
8. Find the average value of f (x, y) = x cos (xy) over the rectangle $0 \le x \le \pi, 0 \le y \le 1$.
9. Find the gradient field of $\phi = xyz$.
10. State Stoke's theorem. **(Weightage 6×1=6)**

P.T.O.

Answer **any seven** from the following **(Weightage 2 each) :**

11. Find the vector projection of $\vec{b} = 6\hat{i} + 3\hat{j} + 2\hat{k}$ onto $\vec{a} = \hat{i} - 2\hat{j} - 2\hat{k}$ and scalar component of $\vec{b}$ in the direction of $\vec{a}$.

12. Find the distance from the point (1, 1, 5) to the line $x = 1 + t$, $y = 3 - t$, $z = 2t$.

13. Find the unit tangent vector and principal unit normal for the circular motion $\vec{r}(t) = \cos 2t\,\hat{i} + \sin 2t\,\hat{j}$.

14. Find $\lim_{(x,y)\to(0,0)} \dfrac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$

15. Find the linearization of $f(x, y) = x^2 - xy + \frac{1}{2}y^2 + 3$ at the point (3, 2).

16. Find the derivative of $f(x, y) = xe^y + \cos(xy)$ at (2, 0) in the direction of the vector $3\hat{i} - 4\hat{j}$.

17. Find the centroid of the region in the first quadrant that is bounded above the line $y = x$ and below the parabola $y = x^2$.

18. Find the area of the region that lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$.

19. Show that the work done by force field $\vec{F} = yz\hat{i} + xz\hat{j} + xy\hat{k}$ is independent of path. Also find the work done along any smooth curve joining the point (–1, 3, 9) to ( 1, 6, –4).

20. Integrate xyz over the surface of the cube cut from the first octant by the planes $x = 1$, $y = 1$ and $z = 1$. **(Weightage 7×2=14)**



Answer **any three** from the following **(Weightage 3 each) :**

21. Find the greatest and smallest values that the function $f(x, y) = xy$ takes on the ellipse $\dfrac{x^2}{8} + \dfrac{y^2}{2} = 1$.

22. Find the volume of the upper region D cut from the solid sphere $\rho \le 1$ by the cone $\phi = \dfrac{\pi}{3}$.

23. Evaluate $\int_0^1 \int_0^{1-x} \sqrt{x + y}\,(y - 2x)^2\, dy\, dx$.

24. Find the flux of $\vec{F} = yz\hat{i} + x\hat{j} - z^2\hat{k}$ outward through the parabolic cylinder $y = x^2, 0 \le x \le 1, 0 \le z \le 4$.

25. Verify Green's theorem in the plane for the field $\vec{F} = (x - y)\hat{i} + x\hat{j}$ and the region R bounded by the unit circle $C : \vec{r}(t) = \cos t\hat{i} + \sin t\hat{j}, 0 \le t \le 2\pi$. **(Weightage 3×3=9)**

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