Reg. No. : ....

Name : ....

K19U 2257

V Semester B.Sc. Degree (CBCSS- Reg./Sup./Imp.)

Examination, November-2019

(2014 Admn. Onwards)

Core Course in Mathematics

5B08 MAT: Vector Calculus

Time : 3 hrs Max. Marks : 48

SECTION - A

All the **4** questions are compulsory. They carry **1** mark each. **(4×1=4)**

1. Find the divergence of $e^x\left(\cos y\, \vec{i} + \sin y\, \vec{j}\right)$.

2. Express $\frac{\partial w}{\partial r}$ in terms of r and s if $w = x + y, x = r + s, y = r - s$.

3. What do you mean by a potential function for a vector field F.

4. Give a parametrization of the cone $z = \sqrt{x^2 + y^2}, 0 \le z \le 1$.

SECTION - B

Answer any **8** questions among the questions 5 to 14. These questions carry **2** marks each. **(8×2=16)**

5. Find the angle between the planes $3x - 6y - 2z = 15$ and $2x + y - 2z = 5$.

6. Show that $\vec{r}(t) = \cos t\,\vec{i} + \sqrt{5}\,\vec{j} + \sin t\,\vec{k}$ has constant length and is orthogonal to its derivative.

7. Define saddle point.

P.T.O.

8. Find the curl with respect to the right hand Cartesian coordinates of $yz\vec{i} + 3zx\vec{j} + z\vec{k}$.

9. Prove that for any twice continuously differentiable scalar function $f, curl(grad\ f) = \vec{0}$.

10. Find the local extreme values of the function $f(x,y) = xy - x^2 - y^2 - 2x - 2y + 4$.

11. Show that $\vec{F} = (2x-3)\vec{i} - z\vec{j} + \cos z\vec{k}$ is not conservative.

12. Evaluate $f(x,y,z) = 3x^2 - 2y + z$ over the line segment C joining the origin to the point $(2,2,2)$.

13. Find the circulation of the field $F = (x-y)\vec{i} + x\vec{j}$ around the circle $\vec{r}(t) = (\cos t)\vec{i} + (\sin t)\vec{j}, 0 \le t \le 2\pi$.

14. Use Green's theorem to find the outward flux for the field $F = (x-y)\vec{i} + (y-x)\vec{j}$ across the curve square bounded by $x = 0, x = 1, y = 0, y = 1$.

SECTION - C

Answer any **4** questions among the questions **15 to 20**. These questions carry **4** marks each. **(4×4=16)**

15. Find and graph the osculating circle for a parabola $y = x^2$ at the origin.

16. Find the distance from $S(1,1,3)$ to the plane $3x + 2y + 6z = 6$.

17. Find the derivative of $f(x,y,z) = x^3 - xy^2 - z$ at $P_0(1,1,0)$ in the direction of $\vec{A} = 2\vec{i} - 3\vec{j} + 6\vec{k}$. Find the direction in which f increases most rapidly at P.

18. Use Taylor's formula to find a quadratic approximation of $f(x,y) = \cos x \cos y$ at the origin. Estimate the error in the approximation if $|x| \le 0.1$ and $|y| \le 0.1$.

19. Integrate $g(x,y,z) = x + y + z$ over the surface of the cube cut from the first octant by the planes $x = a, y = a, z = a$.

20. Find the surface area of a sphere of radius a.

SECTION - D

Answer any **2** questions among the questions 21 to 24. These questions carry **6** marks each. **(2×6=12)**

21. Find:
   a) Unit tangent vector T,
   b) Unit normal vector N,
   c) Curvature K,
   d) Torsion τ and binomial vector B for the space curve $\vec{r}(t) = (3\sin t)\vec{i} + 3(\cos t)\vec{j} + 4t\vec{k}$.

22. Find the absolute maximum and minimum values of $f(x,y) = 2 + 2x + 2y - x^2 - y^2$ on the triangular plate bounded by the lines $x = 0, y = 0, y = 9 - x$.

23. a) State both forms of Green's theorem.
   b) Verify the circulation -curl form of Green's theorem for the field $\vec{F}(x,y) = (x-y)\vec{i} + x\vec{j}$ and the region R bounded by the unit circle.

   $C : \vec{r}(t) = \cos t\vec{i} + \sin t\vec{j}, 0 \le t \le 2\pi$

24. a) State Stoke's theorem.
   b) Use Stoke's theorem to evaluate $\int_c \vec{F}.d\vec{r}$, if $F = xz\vec{i} + xy\vec{j} + 3xz\vec{k}$ C is the boundary of the portion of the plane $2x + y + z = 2$ in the first octant, traversed counter clock wise.
