K18U 1477

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS-Reg./Sup./Imp.) Examination, November 2018
(2014 Admn. Onwards)
CORE COURSE IN MATHEMATICS
5B07 MAT-Differential Equations, Laplace Transform and Fourier Series

Time : 3 Hours Max. Marks : 48

SECTION – A

All the first **4** questions are **compulsory**. They carry 1 mark **each**. **(4×1=4)**

1. Solve $y' + 3x^2y^2 = 0$.
2. Find the eigenvalues of the matrix $A = \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}$
3. Define even function. If f(x) is an even function, show that
$$\int_{-L}^{L} f(x)\,dx = 2\int_{0}^{L} f(x)\,dx.$$
4. Let f(x) is a p periodic function. Show that f(x) is np periodic for n = 2, 3, ...

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry 2 marks **each**. **(8×2=16)**

5. State existence and uniqueness theorem for the initial value problem $y' = f(x, y) \quad y(x_0) = y_0$.
6. Show that e^x is an integrating factor of the differential equation $\sin y dx + \cos y dy = 0$ and hence solve it.
7. Define Wronskian of two functions f, g. Show that if f, g are linearly dependent then W(f, g) = 0.

8. Find the general solution of $x^2y'' + xy' + y = 0$.

9. Solve $y'' + 2ky' + (k^2 + 4)\,y = 0$.

10. Solve $y'' - 3y' + 2y = 2e^x + 6e^{2x}$.

11. Find the inverse Laplace transform of $\frac{s-4}{s^2+4}$.

12. Find the Laplace transform of coshat.

13. Find the Fourier integral representation of the function $f(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$.

14. If L(f) denote the Laplace transform of the function f(x). Show that

$L(f_1 + f_2) = L(f_1) + L(f_2), \quad L(cf) = cL(f)$.

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry 4 marks **each**. **(4×4=16)**

15. Find the general solution of the differential equation $y' + xy = 4x$.

16. Verify that $-\frac{1}{2}e^x \sin x$ is a solution of the differential equation $(D^2 - 4D + 4)y = e^x \cos x$ and find a general solution.

17. Find the inverse transform of the function $\ln\left(1+\frac{\omega^2}{s^2}\right)$.

18. If L(f) denote the Laplace transform of the function f(x). Find f such that $L(f) = \frac{s}{(s+1)^2}$.

19. Find the Fourier series of the function

$$f(x) = \begin{cases} 0 & \text{if } -2 < x < -1 \\ k & \text{if } -1 < x < 1 \\ 0 & \text{if } 1 < x < 2 \end{cases}$$

20. Find the Fourier coefficients of the 2π periodic function defined by $f(x) = x + |x|,\ -\pi < x < \pi$.

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**. **(2×6=12)**

21. Find the orthogonal trajectories of the family of curves $xy = c$.

22. Solve the initial value problem

$y'' - 3y' + 2y = 12x^2 + 6x^3 - x^4,\ y(0) = 4,\ y'(0) = -8$.

23. Using Laplace transform solve the integral equation

$$y = 2t - 4\int_0^t y(\tau)(t-\tau)\,d\tau.$$

24. Find the Fourier series of the following periodic function f(x) of period $p = 2L$ defined by $f(x) = x + x^2,\ -1 < x < 1$.

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