



K19U 2256



Reg. No. :

Name :

V Semester B.Sc. Degree (CBCSS- Reg./Sup./Imp.)

Examination, November-2019

(2014 Admn. Onwards)

Core Course in Mathematics

5B 07 MAT: Differential Equations, Laplace Transform and Fourier series

Time : 3 Hours

Max. Marks : 48

SECTION - A

All the 4 questions are compulsory. They carry 1 mark each. (4x1=4)

1. Solve the differential equation $y'' = x^{-4}$.
2. Evaluate $(D-2)(D+1)e^{2x}$
3. Find the Laplace transform of $e^t \cosh 3t$
4. Show that if $f(x)$ and $g(x)$ have period p , then $h = af + bg$, where a and b are constants, has period p .

SECTION - B

Answer any 8 questions among the questions 5 to 14. These questions carry 2 marks each. (8x2=16)

5. Show that $2xy dx + x^2 dy = 0$ is exact and hence solve it.
6. Solve $y' - y = e^{2x}$.

P.T.O.



7. Solve the boundary value problem $y'' + y = 0, y(0) = 3, y(\pi) = -3$.
8. Define the Wronskian of two solutions y_1, y_2 of second order linear homogenous equation and find the Wronskian of e^x and xe^x .
9. Solve the non homogenous equation $y'' + 4y = 8x^2$.
10. Find a basis of solutions for $x^2y'' - xy' + y = 0$, for positive x .
11. Define the unit step function and derive its Laplace transform.
12. State the convolution theorem and find the convolution of 1 and t .
13. Find the Fourier series of $f(x) = x + \pi$ if $-\pi < x < \pi$ and $f(x + 2\pi) = f(x)$.
14. State the Fourier convergence theorem.

SECTION - C

Answer any 4 questions among the questions 15 to 20. These questions carry 4 marks each. (4×4=16)

15. Give an example of an initial value problem, which has more than one solution.
16. State and prove the superposition principle for the homogenous linear system.
17. Solve $y'' + 10y' + 25y = e^{-5x}$.
18. Factor $p(D) = D^2 + D - 6$ and solve $p(D)[y] = 0$.
19. Find the inverse Laplace transform of $F(s) = \frac{2}{s^2} - \frac{2e^{-2s}}{s^2} - \frac{4e^{-2s}}{s} + \frac{se^{-\pi s}}{s^2 + 1}$.



20. Find the Fourier series of $f(x) = |x|, -2 < x < 2, f(x+4) = f(x)$.

SECTION - D

Answer any 2 questions among the questions 21 to 24. These questions carry 6 marks each. (2×6=12)

21. Find the orthogonal trajectories of $y = cx^2$, where c is arbitrary.
22. Solve the differential equation $y'' + y = \sec x$.
23. Find the solution of $y'' + 2y' + 2y = 5u(t - 2\pi)\sin t, y(0) = 1, y'(0) = 0$.
24. Find the Fourier series of $f(x) = \frac{x^2}{2}, -\pi < x < \pi, f(x + 2\pi) = f(x)$. Hence show

$$\text{that } 1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \dots = \frac{\pi^2}{12} \text{ and } 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \dots = \frac{\pi^2}{6}.$$