K20U 0468

Reg. No. : .....

Name : .....

II Semester B.Sc. Degree (CBCSS (OBE) – Regular) Examination, April 2020
(2019 Admission)
Core Course in Mathematics
2B02 MAT : INTEGRAL CALCULUS AND LOGIC

Time : 3 Hours Max. Marks : 48

PART – A

Short Answer Questions – Answer **any 4**.

1. Evaluate $\int_0^{\frac{\pi}{2}} \sin^5 x \, dx$.
2. Express the equation $x + y = 0$ in polar form.
3. Express the cartesian coordinates (x, y, z) in terms of the spherical coordinates (ρ,θ,ϕ).
4. Give an example for a sentence which is not a statement.
5. Check the truthness/falsity of the statement $(\forall n \in \mathbb{P})\ (n + 2 > 3)$, where $\mathbb{P}$ is the set of all natural numbers. Justify your answer. **(4×1=4)**

PART – B

Short Essay Questions – Answer **any 8**.

6. Find $\frac{d}{dx}(\text{sech}^2(x^3))$.
7. Evaluate $\int e^{-x} \sinh 2x \, dx$.
8. Evaluate $\int_0^3 \int_0^4 \frac{e^{xy}}{y} dydx$.
9. Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{2\cos\theta} r \, drd\theta$.
10. Find the area of a disc of radius a using double integral.

P.T.O.

11. Find $\int_0^4 x^2\,dx$ using trapezoidal rule, taking two subintervals.

12. Find $\int_0^4 f(x)\,dx$ using Simpson's 1/3 rule, where the function f(x) is given by

x	0	1	2	3	4
f(x)	1	2	4	7	9

13. Explain the terms :
    i) Conjunction of two statements
    ii) Contrapositive of an implication.

14. If n is an odd integer, prove that $n^2 + 1$ is an even integer.

15. Write the converse and contrapositive of the statement : if $x \geq y$, then $x - y < 0$.

16. Show that if m and n are natural numbers and $m + n = 10$, then either $m \leq 5$ or $n \leq 5$.

(8×2=16)

PART – C

Essay Questions – Answer **any 4**.

17. Evaluate $\int_0^{\frac{\pi}{4}} \sin^3 4\theta \cos^2 2\theta\, d\theta$.

18. Derive the reduction formula for $\int \tan^n x\,dx$, $n > 1$.

19. Convert into polar form and evaluate $\int_0^3 \int_x^3 \frac{y^2}{x^2+y^2}\,dydx$.

20. Evaluate $\int_0^1 \int_0^{2x} \int_{x-z}^{x+z} (x^2 + 2z)\,dV$.

21. Explain the Simpson's 1/3 rule of integration.

22. Prove the following statement using the method of contradiction : "If n is an integer and n^2 is odd, then n must be odd". Is the converse true ? Justify.

23. Write the negation of the following statements using quantifiers.
    i) $(\forall x \in \mathbb{R})\ (2x < 7)$
    ii) $(\exists x \in \mathbb{R})\ (2x > 7)$.

(4×4=16)

PART – D

Long Essay Questions – Answer **any 2**.

24. Derive reduction formula for $\int \sin^m x \cos^n x\,dx$, where m, $n > 1$ and use it to evaluate $\int_0^{\frac{\pi}{2}} \sin^4 x \cos^6 x\,dx$.

25. Use triple integration in spherical coordinates to find the volume of the solid V bounded above by the sphere $x^2 + y^2 + z^2 = 4$ and bounded below by the cone $z = \sqrt{x^2+y^2}$.

26. Evaluate $\int_0^6 \frac{1}{2x+1}\,dx$ using Simpson's 1/3 rule, taking six subintervals.

27. a) Prove that if x is a real number greater than 1, than $x^2 > x$.

    b) Test the truthness of the following statements (with justifications) :
    i) $(\forall x \in \mathbb{R})\ (x^2 \geq x)$, where $\mathbb{R}$ is the set of all real numbers.
    ii) $(\forall x \in \mathbb{Z})\ (x^2 \geq x)$, where $\mathbb{Z}$ is the set of all integers.

(2×6=12)