K19U 2255

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS- Reg./Sup./Imp.) Examination, November - 2019

(2014 Admn. Onwards)

Core Course in Mathematics

5B 06 MAT: ABSTRACT ALGEBRA

Time : 3 Hours Max. Marks : 48

SECTION - A

Answer **All** Questions, Each question carries **One** Mark. **(4×1=4)**

1. Is the usual addition a binary operation on the set of all prime numbers? Justify your answer.

2. Define orbits of a permutation σ of a set A.

3. Define normal subgroup of a group G. Given an example.

4. What is the characteristic of the ring of real numbers under usual addition and multiplication?

SECTION - B

Answer any **Eight** Questions, Each question carries **Two** Marks. **(8×2=16)**

5. Prove that every cyclic group is abelian.

6. Show that a nonempty subset H of group G is a subgroup of G if and only if $ab^{-1} \in H$ for all $a, b \in H$.

7. Describe S_n, the symmetric group on n letters.

8. Find the index of <3> in the group Z_{24}.

P.T.O.

9. Prove that the identity permutation in S_n is an even permutation for $n \geq 2$.

10. Determine the number of group homomorphisms from z onto z.

11. Find the characteristic of the ring $Z_6 \times Z_{21}$.

12. Prove that every field is an integral domain.

13. Define a ring and give an example of a finite ring which is not an integral domain.

14. Find the remainder of 8^{103} when divided by 13.

SECTION - C

Answer any **Four** Questions, Each question carries **four** Marks.**(4×4=16)**

15. Show that every finite cyclic group of order n is isomorphic to $< Z_n, +_n >$.

16. Show that the set of all permutations of any nonempty set A is group under permutation multiplication.

17. Prove that every group of prime order is cyclic.

18. Let φ be a homomorphism of a group G into G'. Then prove the following:

    a) If $a \in G$, prove that $\varphi(a^{-1}) = (\varphi(a))^{-1}$

    b) If H is a subgroup of G, then $\phi[H]$ is a subgroup of G'

19. Prove that every finite integral domain is a field.

20. Prove that in the ring Z_n the divisors of 0 are precisely those nonzero elements that are not relatively prime to n.

SECTION - D

Answer any **Two** Questions, Each Question carries **Six** Marks.**(2×6=12)**

21. a) Define the greatest common divisor of two positive integers. Also find the quotient and remainder when 50 is divided by 8 according to division algorithm.

    b) Prove that subgroup of a cyclic group is cyclic.

22. a) State and prove Lagrange's Theorem.

    b) Prove that the collection of all even permutations of $\{1,2,3,\ldots\ldots\ldots,n\}$ forms a subgroup of order $\frac{n!}{2}$ of the symmetric group $S_n; n \geq 2$.

23. a) State and prove the fundamental homomorphism theorem.

    b) Show that a group homomorphism is one-one if and only if its kernel consists of only the identity element.

24. a) Show that the cancellation law holds in a ring iff it has no divisors of 0.

    b) Show that the matrix $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ is a divisor of 0 in $M_2(z)$.