



Reg. No. :

Name :



K17U 1696

V Semester B.Sc. Degree (CBCSS – Reg./Sup./Imp.)
 Examination, November 2017
 (2014 Admn. Onwards)
 CORE COURSE IN MATHEMATICS
 5B06 MAT : Abstract Algebra

Time : 3 Hours

Max. Marks : 48

SECTION – A

Answer **all** the questions. **Each** question carries **one** mark.

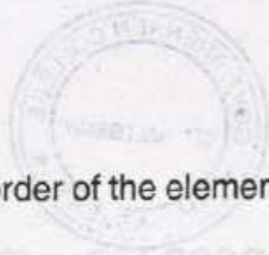
1. Find the number of elements in the cyclic subgroup of \mathbb{Z}_{30} generated by 25.
2. What are the orbits of the identity permutation σ of a set A ?
3. How many homomorphisms are there of \mathbb{Z} onto \mathbb{Z} ?
4. What are the units in $\mathbb{Z} \times \mathbb{Z}$? (4×1=4)

SECTION – B

Answer **any 8** questions. **Each** question carries **two** marks.

5. Show that every permutation of a finite set is a product of disjoint cycles.
6. Let H be a subgroup of a finite group G. Show that the order of H is a divisor of the order of G.
7. Show that if σ is a cycle of odd length, then σ^2 is a cycle.
8. Find all cosets of the subgroup $\langle 2 \rangle$ of \mathbb{Z}_{12} .
9. Express the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 8 & 2 & 6 & 3 & 7 & 4 & 5 & 1 \end{pmatrix}$ as a product of disjoint cycles and then as a product of transpositions.

P.T.O.



10. Determine the order of the element $5 + \langle 4 \rangle$ in $\mathbb{Z}_{12}/\langle 4 \rangle$.
11. Show that any group homomorphism $\phi: G \rightarrow G'$ where $|G|$ is a prime must either be the trivial homomorphism or a one-to-one map.
12. Show that in the ring \mathbb{Z}_n , the divisors of 0 are precisely those nonzero elements that are not relatively prime to n .
13. Show that every finite integral domain is a field.
14. If $a \in \mathbb{Z}$ and p is a prime not dividing a , show that p divides $a^{p-1} - 1$. (8×2=16)

SECTION - C

Answer any 4 questions. Each question carries four marks.

15. Show that every subgroup of a cyclic group is cyclic.
16. State and prove Lagrange's theorem. Deduce that the order of an element of a finite group divides the order of the group.
17. Let G and G' be groups and let $\phi: G \rightarrow G'$ be a one-to-one function such that $\phi(xy) = \phi(x)\phi(y)$ for all $x, y \in G$. Show that $\phi[G]$ is a subgroup of G' and ϕ provides an isomorphism of G with $\phi[G]$.
18. Let $\phi: G \rightarrow H$ be a group homomorphism. Show that $\phi[G]$ is abelian if and only if for all $x, y \in G$ we have $xyx^{-1}y^{-1} \in \text{Ker}(\phi)$.
19. Describe all ring homomorphisms of \mathbb{Z} into $\mathbb{Z} \times \mathbb{Z}$.
20. Find all solutions of the congruence $15x \equiv 27 \pmod{18}$. (4×4=16)



SECTION - D

Answer any 2 questions. Each question carries six marks.

21. If a is a generator of a finite cyclic group G of order n , show that the other generators of G are the elements of the form a^r , where r is relatively prime to n .
22. List the elements of the symmetric group S_3 on 3 letters and form the multiplication table for S_3 . Find all subgroups of S_3 .
23. State and prove the fundamental homomorphism theorem.
24. a) Show that the cancellation laws hold in a ring R if and only if R has no divisors of 0.
b) Show that $a^2 - b^2 = (a + b)(a - b)$ for all a and b in a ring R if and only if R is commutative.
c) Show that 1 and $p - 1$ are the only elements of the field \mathbb{Z}_p that are their own multiplicative inverse. (2×6=12)