



M 7150

Reg. No. :

Name :

V Semester B.Sc. Degree (CCSS – Reg./Supple./Imp.)
 Examination, November 2014
 CORE COURSE IN MATHEMATICS
 5B07 MAT : Abstract Algebra

Time: 3 Hours

Max. Weightage : 30

1. Mark **each** of the following **true** or **false** :

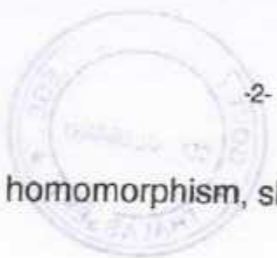
- a) A binary operation on a set S assigns at least one element of S to each ordered pair of elements of S.
- b) In a group each linear equation has a solution.
- c) Every cyclic group is abelian.
- d) Any group of prime order is cyclic.

(Wt. 1)

Answer **any six** questions from the following (Weightage **one each**) :

- 2. If $(a * b)^2 = a^2 * b^2$ for a and b in a group G, show that $a * b = b * a$, where $a^2 = a * a$.
- 3. Prove that every cyclic group is abelian.
- 4. Obtain the group of symmetries of an equilateral triangle with vertices 1, 2 and 3.
- 5. Write the permutation $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 8 & 6 & 7 & 4 & 1 & 5 & 2 \end{pmatrix}$ as a product of cycles.
- 6. If G is a group and H is a subgroup of G, prove that the relation '~' defined on G by $a \sim b$ if and only if $a^{-1}b \in H$ is an equivalence relation.

P.T.O.



7. If $\varphi: G \rightarrow G'$ is a group homomorphism, show that $\text{Ker } \varphi$ is a normal subgroup of G .
8. Prove that a factor group of a cyclic group is cyclic.
9. Define a ring homomorphism. Check whether $\varphi: \mathbb{Z} \rightarrow \mathbb{Z}$ defined by $\varphi(x) = 2x$ is a ring homomorphism.
10. Solve the equation $x^2 - 5x + 6 = 0$ in \mathbb{Z}_{12} .
11. Show that \mathbb{Z}_p has no zero divisors if p is a prime. (6×1=6)

Answer **any seven** questions from the following (Weightage **2 each**):

12. In a group G , prove that there is only one identity element and also prove that the inverse of every element is unique.
13. Prove that the intersection of two subgroups H and K of a group G is a subgroup of G .
14. If G is a group and $a \in G$, show that $H = \{a^n/n \in \mathbb{Z}\}$ is the smallest subgroup of G that contains 'a'.
15. If H is a subgroup of a finite group G , prove that order of H is a divisor of order of G . Also prove that the order of an element of a finite group divides the order of the group.
16. If A is a nonempty set and S_A is the collection of all permutations of A , prove that S_A is a group under permutation multiplication.
17. Define a homomorphism of a group G into a group G' . If $\varphi: G \rightarrow G'$ is a homomorphism of a group G onto a group G' and G is abelian, show that G' is also abelian.
18. Show that the mapping $\varphi: S_n \rightarrow \mathbb{Z}_2$ defined by
- $$\varphi(\sigma) = \begin{cases} 0 & \text{if } \sigma \text{ is an even permutation} \\ 1 & \text{if } \sigma \text{ is an odd permutation} \end{cases}$$
- is a homomorphism, where S_n is the symmetric group of n letters and $\sigma \in S_n$.



19. Prove that a group homomorphism $\varphi: G \rightarrow G'$ is a one-to-one map if and only if $\text{Ker } \varphi = \{e\}$, where e is the identity element of G .
20. Prove that the cancellation law hold in a ring R if and only if R has no zero divisors.
21. Show that every field is an integral domain. (7×2=14)

Answer **any three** questions from the following (Weightage **3 each**):

22. Prove that a subgroup of a cyclic group is cyclic.
23. Show that every group is isomorphic to a group of permutations.
24. Prove that no permutation in S_n can be expressed both as a product of even number of transpositions and as a product of an odd number of transpositions.
25. If φ is a homomorphism from a group G into a group G' , prove the following:
- $\varphi(e)$ is the identity element of G' , where e is the identity element of G .
 - $\varphi(a^{-1}) = \varphi(a)^{-1}$, $a \in G$
 - $\varphi[H]$ is a subgroup of G' , where H is a subgroup of G .
 - $\varphi^{-1}[K']$ is a subgroup of G , where K' is a subgroup of G' .
26. Show that every finite integral domain is a field. (3×3=9)