GOVT. BRENNEN COLLEGE CENTRAL LIBRARY THALASSERY

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS – Reg./Sup./Imp.) Examination, November 2020
(2014 Admn. Onwards)
CORE COURSE IN STATISTICS
5B06STA : Mathematical Analysis – I

Time : 3 Hours Max. Marks : 48

Instruction : *Use of Calculators and Statistical Tables are permitted.*

PART – A
(Short Answer)

Answer **all** the questions : **(6×1=6)**

1. Define limit of a sequence.
2. Determine the bounds of the sequence $\{1 - (-1)^n\}$n is a positive integer.
3. State Cauchy's root test.
4. Define continuity of a function at a point.
5. Find $\lim_{x \to \infty} \frac{x^2 - 3x + 2}{3x^2 - 1}$.
6. Check the differentiability of the function $f(x) = |x|$ at the point $x = 0$.

PART – B
(Short Essay)

Answer **any seven** questions : **(7×2=14)**

7. Show that every convergent sequence is bounded.
8. Find $\lim_{x \to 0} \frac{e^{3x} - 1}{x}$.

9. Define a Cauchy's sequence.

10. Distinguish between absolute and conditional convergence.

11. Test the convergence of the infinite series whose n^{th} term is $\frac{1.2.3....n}{7.10....(3n+4)}$.

12. What are the different types of discontinuities ? Explain.

13. Show that the function $f(x) = \begin{cases} x, & \text{when x is rational} \\ 0, & \text{when x is irrational} \end{cases}$ is discontinuous at every point.

14. Expand e^x using Maclaurins theorem.

15. State Darboux's theorem.

PART – C

(Essay)

Answer **any four** questions : **(4×4=16)**

16. If $\{a_n\}$ and $\{b_n\}$ are two sequences with limits a and b respectively, show that $\lim(a_n + b_n) = a + b$.

17. Show that the sequence $\{b_n\}$ where $b_n = \left[\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + ... + \frac{1}{(2n)^2}\right]$ converges to zero.

18. Test for the convergence of the series with n^{th} term $(n^3+1)^{1/3} - n$, using comparison test.

19. Show that limit of a function is unique.

20. Show that uniform continuity of a function implies continuity.

21. Discuss the validity of Rolle's theorem for the function $f(x) = 2x^4 + 3x^2 - 1$ defined in $[-2, 2]$.

PART – D

(Long Essay)

Answer **any two** questions : **(2×6=12)**

22. Evaluate :

a) $\lim_{n\to\infty}\left(1 \cdot \frac{2}{1} \cdot \frac{3}{2} \cdots \frac{n}{n-1}\right)^{1/n}$

b) $\lim_{n\to\infty} \frac{1+2+3+...+n}{n^2}$.

23. Show that every absolutely convergent is convergent. Discuss the convergence of $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ...$

24. Prove that if a function is continuous in a closed interval it is bounded therein.

25. State and prove Lagrange's mean value theorem. Examine the theorem for the function $f(x) = \log x$ on $\left[\frac{1}{2}, 2\right]$.