M 7151

Reg. No. : ...............................

Name : ...............................

**V Semester B.Sc. Degree (CCSS – Reg./Supple./Imp.)**
**Examination, November 2014**
**CORE COURSE IN MATHEMATICS**
**5 B08 MAT : Graph Theory**

Time : 3 Hours Max. Weightage : 30

***Instruction :*** *Answer to **all** questions.*

Fill in the blanks :

1. a) The number of edges in the graph Km, n is ______________

   b) The complete graph Ka has ______________ different spanning trees.

   c) The complete bipartite graph $K_{n,n}$ on $2_n$ vertices is ______________ regular.

   d) Suppose G is a graph with n vertices. Then the order of the adjacency matrix of G is ______________ (Wt. 1)

Answer **any six** from the following. Wt. **1 each**.

2. Define "underline simple graph" of a graph G with an example.

3. Draw the join of the graph $K_1$ and $K_2$.

4. Define 'cut vertex' of a graph G with an example.

5. Define Euler and Hamiltonian graphs.

6. Define perfect matching in a graph G with an example.

7. When a digraph D is said to be strongly connected give an example.

P.T.O.

8. Draw the de-Bruijin diagram $D_{2,3}$.

9. Prove that if a Tournament T is strongly connected then it is Hamiltonian.

10. Define the square of simple connected graph G with an example. **(Wt. 6×1=6)**

Answer **any seven** of the following. Wt. **2 each.**

11. Prove that in any graph G there is an even number of odd vertices.

12. Let G be a graph with n vertices and Let A denote the adjacency matrix of G. Let $B = (b_{ij})$ be the matrix $B = A + A^2 + ... + A^{n-1}$. Prove that G is connected iff B has no zero entries off the main diagonal.

13. Let G be an acyclic graph with n vertices and k connected components. Then prove that G has $(n - k)$ edges.

14. Let G be a connected graph. Then prove that G is a tree if and only if every edge of G is bridge.

15. Let v be a vertex of a connected graph G. Then prove that v is a cut vertex of G if and only if there are two vertices u and w of G, both different from v such that v is on every $u - w$ path in G.

16. It G be a graph in which the degree of every vertex is at least two. Then prove that G contains a cycle.

17. Prove that a simple graph G is Hamiltonian if and only if its closure c(G) is Hamiltonian.

18. Prove that a matching H in a graph G is a maximum matching if G contains no H – augmenting path.

19. Prove that for each pair of positive integer n and k, both greater than one, the de-Bruijin diagram $D_n$, k has a directed Euler tour.

20. Prove that every tournament T has a directed Hamiltonian path. **(Wt. 7×2=14)**



Answer **any 3** of the following. Wt. **3 each**.

21. Let G be a non-empty graph with at least two vertices. Then prove that G is bipartite if and only if it has no odd cycle.

22. Let e be an edge of the graph G and Let G-e be the subgraph obtained by deleting e. Then prove that $W(G) \leq W(G - e) \leq W(G) + 1$.

23. Prove that a connected graph G is Euler if and only if the degree of every vertex is even.

24. Let D be a weakly connected diagraph with atleast one arc. Then prove that D is Euler if and only if od (v) = id (v) for every vertex v of D.

25. Prove that A a graph G is orientable if and only if it is connected and has no bridges. **(Wt. 3×3=9)**