



K17U 1699

Reg. No. :

Name :

V Semester B.Sc. Degree (CBCSS-Reg./Sup./Imp.) Examination,
November 2017

(2014 Admn. Onwards)

CORE COURSE IN MATHEMATICS

5B 09 MAT : Graph Theory

Time : 3 Hours

Max. Marks : 48

SECTION – A

Answer **all** the questions. **Each** question carries **one** mark.

1. What is the smallest integer n such that the complete graph with n vertices has at least 500 edges ?
2. Give two nonisomorphic simple connected graphs G and G' such that their line graphs are isomorphic.
3. For what values of n , a complete graph on n vertices is Eulerian ?
4. Draw a simple graph which is Eulerian but not Hamiltonian. (4×1=4)

SECTION – B

Answer **any 8** questions. **Each** question carries **two** marks.

5. If G is simple and $\delta \geq \frac{n-1}{2}$ then show that G is connected.
6. Show that an edge $e = xy$ is a cut edge of a connected graph G if and only if there exist vertices u and v such that e belongs to every $u - v$ path in G .
7. Show that a graph G with at least three vertices is 2-connected if and only if any two vertices of G lie on a common cycle.
8. Show that a simple graph is a tree if and only if any two distinct vertices are connected by a unique path.
9. Prove that every tree is a bipartite graph.
10. Show that a simple connected graph contains at least $m - n + 1$ distinct cycles.

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K17U 1699



11. What are central vertices of a graph G ? Give an example of a tree with two central vertices.
12. Show that a connected graph G is a tree if and only if every edge of G is a cut edge of G .
13. Show that a subset S of the vertex set V of a graph G is independent if and only if $V \setminus S$ is a covering of G .
14. Let D be a digraph with no directed cycle. Prove that there exists a vertex whose indegree is 0. (8×2=16)

SECTION – C

Answer **any 4** questions. **Each** question carries **four** marks.

15. a) Show that the line graph of a simple graph G is a path if and only if G is a path.
b) Illustrate the join of two vertex-disjoint graphs with an example.
16. Prove that in a connected graph with at least three vertices, any two longest paths have a vertex in common.
17. If C is any cycle of a simple block G with at least three vertices, show that there exists a sequence of nonseparable subgraphs $C = B_0, B_1, \dots, B_r = G$ such that B_{i+1} is an edge-disjoint union of B_i and a path P_i , where the only vertices common to B_i and P_i are the end vertices of P_i , $0 \leq i \leq r-1$.
18. Show that every Eulerian graph has an odd number of cycle decompositions.
19. Let G be simple graph with $n \geq 3$ vertices. If for every pair of nonadjacent vertices u, v of G , $d(u) + d(v) \geq n$, show that G is Hamiltonian.
20. Show that every tournament contains a directed Hamilton path. (4×4=16)

SECTION – D

Answer **any 2** questions. **Each** question carries **six** marks.

21. Show that a graph is bipartite if and only if it contains no odd cycles.
22. Show that the connectivity and edge connectivity of a simple cubic graph are equal.
23. For any graph G for which $\delta > 0$, show that $\alpha' + \beta' = n$.
24. Show that every vertex of a disconnected tournament T with $n \geq 3$ vertices is contained in a directed k -cycle, $3 \leq k \leq n$. (2×6=12)