Reg. No. : ....

Name : ....

K19U 2258

V Semester B.Sc. Degree (CBCSS- Sup./Imp.) Examination, November-2019

(2014-2016 Admissions)

Core Course in Mathematics

5B 09 MAT: GRAPH THEORY

Time : 3 Hours Max. Marks : 48

SECTION - A

All the 4 questions are compulsory. They carry 1 mark each. **(4×1=4)**

1. Define self-complementary graphs.
2. State Cayley's formula.
3. What is meant by covering of a graph?
4. Define digraph.

SECTION - B

Answer any 8 questions among the questions 5 to 14. These questions carry 2 marks each. **(8×2=16)**

5. Prove that the sum of the degrees of the vertices of a graph is equal to twice the number of edges.
6. Let G be a simple graph. Prove that if G is disconnected then G^c is connected.
7. Let G be a connected graph and $e = xy$ be a cut edge of G. Prove that e does not belong to any cycle of G.
8. Determine the connectivity and edge-connectivity of the Petersen graph.
9. Prove that a tree with at least two vertices contains at least two pendant vertices.
10. Define branch, weight, centroid vertex of a vertex of a tree.
11. For any graph G with n vertices, prove $\alpha + \beta = n$.
12. Define Hamiltonian graphs and traceable graphs.
13. Define a symmetric digraph with an example.
14. Define in degree and out degree of a digraph with an example.

P.T.O.

SECTION - C

Answer any 4 questions among the questions 15 to 20. These questions carry 4 marks each. **(4×4=16)**

15. Explain with examples any four operations of graphs.

16. Prove that if G is a line graph, then $K_{1,3}$ is a forbidden subgraph of G.

17. Prove that a connected graph with at least two vertices contains at least two vertices that are not cut vertices.

18. Prove that every tree has a center consisting of either a single vertex or two adjacent vertices.

19. Determine the values of the parameters $\alpha, \beta, \alpha', \beta'$ for K_n.

20. Prove that a simple graph G with $n \geq 3$, if $d(u)+d(v) \geq n-1$ for every pair of non adjacent vertices u and v of G, then G is traceable.

SECTION - D

Answer any 2 questions among the questions 21 to 24. These questions carry 6 marks each. **(2×6=12)**

21. a) Define line graphs **(1)**

b) Let G_1 and G_2 be simple graphs. Prove that if G_1 and G_2 are isomorphic then $L(G_1)$ and $L(G_2)$ are isomorphic. **(3)**

c) Does the converse hold? Justify. **(2)**

22. a) Prove that for any loopless connected graph $k(G) \leq \lambda(G) \leq \delta(G)$. **(4)**

b) Determine $k(P)$ and $\lambda(P)$ for the Petersen graph P. **(2)**

23. For any non-trivial connected graph G, prove the following statements are equivalent. **(6)**

i) G is Eulerian.

ii) The degree of each vertex of G is an even positive integer.

iii) G is an edge-disjoint union of cycles.

24. a) Define tournaments and display all tournaments on three vertices. **(2)**

b) Prove that every tournament contains a directed Hamilton path. **(4)**
