K20U 1536

Reg. No. : ....

Name : ....

V Semester B.Sc. Degree (CBCSS-Sup.) Examination, November 2020
(2014 – 16 Admns.)
CORE COURSE IN MATHEMATICS
5B09MAT : Graph Theory

Time : 3 Hours Max. Marks : 48

SECTION – A

All the **first 4** questions are **compulsory**. They carry **1** mark **each**.

1. Define a vertex cut of a graph G.
2. What is meant by weight of a vertex u of a tree T ?
3. When will you say that a graph is Eulerian ?
4. Define vertex independent of graphs. **(4×1=4)**

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. These questions carry **2** marks **each**.

5. Show that the sum of the degrees of the vertices of a graph is equal to twice the number of its vertices.
6. Distinguish between Cartesian product and normal product of graphs.
7. An edge e is a cut edge of a connected graph G, if and only if, e does not belong to any cycle of G. Prove it.
8. Show that a simple graph is a tree if and only if, any two vertices are connected by a unique path.
9. If $\delta(G) \geq 2$, for a graph G, then show that G contains a cycle.
10. If any two vertices of a graph G with at least three vertices, lie on a common cycle, then show that G is 2-connected.

P.T.O.

11. Prove that a subset S of the vertex set V of a graph is independent if V – S is a covering of G.

12. Distinguish between Eulerian and non Eulerian graphs with example.

13. Show that if a connected graph G is Eulerian, then the degree of each vertex is an even positive integer.

14. What is meant by orientation of a digraph ? Explain with an example. **(8×2=16)**

SECTION – C

Answer **any 4** questions from among the questions **15** to **20**. These questions carry **4** marks **each**.

15. Prove that the number of edges of a simple graph with ω components cannot exceed $\frac{(n-\omega)(n-\omega+1)}{2}$.

16. Show that the line graph of a simple graph G is a path if and if, G is a path.

17. Prove that a connected simple graph G is 3-edge connected if and only if, every edge of G is the intersection of the edge set of two cycles of G.

18. Show that every 2k-edge connected graph ($k \geq 1$) contains k pairwise edge disjoint spanning tree.

19. Discuss about vertex independence and edge independence with suitable examples.

20. Define degree of a vertex of a graph. Explain indegree and outdegree of a digraph with example. **(4×4=16)**

SECTION – D

Answer **any 2** questions from among the questions **21** to **24**. These questions carry **6** marks **each**.

21. Prove that if two simple graphs G_1 and G_2 are isomorphic, then $L(G_1)$ and $L(G_2)$ are isomorphic.

22. Prove that the number of edges of a connected graph with n vertices is n – 1 if and only if, it is a tree.

23. Show that for a graph G with $\delta > 0$, $\alpha' + \beta' = n$.

24. Prove that every tournament contains a directed Hamilton path. **(2×6=12)**