



K16U 1105



Reg. No. :

Name :

**II Semester B.Sc. Degree (CCSS – Supple./Improv.) Examination, May 2016
COMPLEMENTARY COURSE IN MATHEMATICS
2C02 MAT : Differential and Integral Calculus
(2013 and Earlier Admn.)**

Time : 3 Hours

Max. Weightage : 30

1. Fill in the blanks :

a) $\frac{d}{dx}(\cosh^{-1} x) = \underline{\hspace{2cm}}$

b) The derivative of $(\log x)^{\log x}$ at $x = e$ is equal to $\underline{\hspace{2cm}}$

c) $\lim_{x \rightarrow 0} \frac{\sin x}{x} = \underline{\hspace{2cm}}$

d) Determine the limit of $\frac{1 + \log x - x}{1 - 2x + x^2}$ ($x \rightarrow 1$). **(Weightage 1)**

Answer **any six** from the following (weightage **1 each**) :

2. Determine the limit of $\frac{e^x - e^{\sin x}}{x - \sin x}$ ($x \rightarrow 0$).

3. Find the volume of the solid obtained by revolving one arc of the cycloid $x = a(\theta + \sin \theta)$, $y = a(1 + \cos \theta)$.

4. State Taylor's theorem.

5. Verify Rolle's theorem for $f(x) = x(x + 3) e^{-\frac{x}{2}}$ in $[-3, 0]$.

6. Prove that the radius of curvature of the catenary $y = \frac{a}{2} \left(e^{\frac{x}{a}} + e^{-\frac{x}{a}} \right)$ is $\frac{y^2}{a}$.



7. If $z = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$. Prove that $\frac{\partial^2 z}{\partial y \partial x} = \frac{x^2 - y^2}{x^2 + y^2}$.

8. Find the area of the loop of the curve $r^2 = a^2 \cos \theta$.

9. Find the surface of the solid formed by revolving the cardioid $r = a(1 + \cos \theta)$ about the initial line.

10. Evaluate $\int_1^{\log 8} \int_0^{\log y} e^{x+y} dx dy$.

(Weightage 6x1=6)

Answer any seven from the following (weightage 2 each) :

11. Evaluate $\iint r^2 \sin \theta dr d\theta$ over the area of the cardioid $r = a(1 - \cos \theta)$ above the initial line.

12. The area included between the curve $y^2 = x^3$ and $x^2 = y^3$ is rotated about the x-axis. Find the volume of the solid generated.

13. Expand $\cos x$ by Maclaurin's series.

14. If $u = (1 - 2xy + y^2)^{\frac{1}{2}}$ show that $x \frac{\partial u}{\partial x} - y \frac{\partial u}{\partial y} = y^2 u^3$.

15. Find the surface generated by the revolution of an arc of the centenary $y = c \cos \frac{x}{c}$ about the axis of x.

16. Integrate : (i) $\sin^3 x dx$, (ii) $\sin^4 x dx$.

17. If $p^2 = a^2 \cos^2 \theta + b^2 \sin^2 \theta$, prove that $\frac{d^2 p}{d\theta^2} = \frac{a^2 b^2}{p^3}$.

18. Find the area in the first quadrant bounded by the x-axis and the curve $x^2 + y^2 = 10$, $y^2 = 9x$.

19. If $\cos^{-1}\left(\frac{y}{b}\right) = \log\left(\frac{x}{n}\right)^n$, prove that $x^2 y_{n+2} + (2n+1)xy_{n+1} + 2n^2 y_n = 0$.

20. Find $\frac{dz}{dt}$ when $z = xy^2 + x^2 y$, $x = at^2$, $y = 2at$.

(Weightage 7x2=14)



Answer any three from the following (weightage 3 each) :

21. Find the volume bounded by the paraboloid $x^2 + y^2 = 16 - z$, the cylinder $x^2 + y^2 = 4$ and the plane $z = 0$.

22. Expand $e^{\sin^{-1} x}$ in powers of x by Maclaurin's theorem and hence obtain the value of e^θ .

23. Show that the evolute of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is $(ax)^{\frac{2}{3}} - (by)^{\frac{2}{3}} = (a^2 + b^2)^{\frac{2}{3}}$.

24. If $u = f(r)$, where $r = \sqrt{x^2 + y^2}$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr}$.

25. Show by double integration that the area lying inside the circle $r = a \sin \theta$

and outside the cardioid $r = a(1 - \cos \theta)$ is $\frac{a^2}{4}(4 - \pi)$. (Weightage 3x3=9)