M 8735

Reg. No. : ....

Name : ....

II Semester B.Sc. Degree (C.C.S.S. – Supple./Improv.) Examination, May 2015
(2013 and Earlier Admn.)
COMPLEMENTARY COURSE IN MATHEMATICS
2C02 MAT : Differential and Integral Calculus

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks : (Weightage :1)

   a) $\frac{d}{dx}\sqrt{(ax^2+2bx+c)}$ = ________

   b) $\frac{d}{dx}(\cosh^{-1}x)$ = ________

   c) $\lim_{x\to 0}(x\log x)$ = ________

   d) Radius of curvature in cartesian form is ________

Answer **any six** from the following (Weightage **1 each**) : **(6×1=6)**

2. If $z=\log\sqrt{x^2+y^2}$, prove that $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=0$.

3. If $y=\cos(m\sin^{-1}x)$ show that $(1-x^2)\,y_{n+2}+(2n+1)\,xy_{n+1}+(m^2-n^2)y_n=0$.

4. Evaluate $\iint r^2\sin\theta\,d\theta\,dr$ over the area of cardioids $r=a(1+\cos\theta)$ above the initial line.

5. Find the volume of the solid obtained by revolving the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ about the axis of x.

6. For the cycloid $x=a(t+\sin t)$, $y=a(1-\cos t)$, prove that $\rho=4a\cos\frac{t}{2}$.

P.T.O.

7. Verify Rolle's theorem for the function $f(x) = x^3 - 4x$, in the interval $[-2, 2]$.

8. Evaluate $\iint r\cos\theta \, dr \, d\theta$ over the region bounded by the semicircle $r = 2\cos\theta$ above the initial line.

9. Find the area bounded by the curves $y^2 = 4 - 2x$ and $y = 2 - x$.

10. Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dxdy$.

Answer **any seven** from the following (Weightage **2 each**) : **(7×2=14)**

11. By changing the order of integration, evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 \, dydx$.

12. Expand $\log x$ in powers of $x - 1$.

13. If $x^y y^x = 1$, find $\frac{dy}{dx}$.

14. Show that for any curve $r = f(\theta)$ the curvature is given by $\left[u + \frac{d^2u}{d\theta^2}\right] \sin^3\varphi$; where $u = \frac{1}{r}$.

15. If $u = \log\left[\frac{(x^4 + y^4)}{(x + y)}\right]$, show by Euler's theorem that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3$.

16. Obtain a reduction formula for $\int \frac{x^n}{(\log x)^m} dx$.

17. Find the area bounded by the curve $xy^2 = 4a^2(2a - x)$ and its asymptote.

18. Find the length of the curve $y = \log\left\{\frac{e^x - 1}{e^x + 1}\right\}$ from $x = 1$ to $x = 2$.

19. Evaluate $\int_0^1 \int_0^1 \frac{dxdy}{\sqrt{1+x^2}\sqrt{1-y^2}}$.

20. Find the volume bounded by the co-ordinate planes and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

Answer **any three** from the following (**Weightage 3 each**) : **(3×3=9)**

21. Find the radius of curvature at any point (x, y) of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

22. Find the volume of the solid generated by the revolution of the tractrix $x = a\cos t + \frac{a}{2}\log\tan^2\frac{t}{2}$, $y = a\sin t$ about its asymptote.

23. Evaluate the surface area of the solid generated by revolving the cycloid $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ about the line $y = 0$.

24. A quadrant of the circle of radius a revolves about its chord. Show that the volume of the spindle generated is $\frac{\pi}{6\sqrt{2}}(10 - 3\pi)a^3$.

25. Show that the n^{th} derivative of $y = \frac{1}{1 + x + x^2 + x^3}$ is $\frac{1}{2}(-1)^n n! \sin^{n+1}\theta\,[\sin(n+1)\theta - \cos(n+1)\theta + (\sin\theta + \cos\theta)^{-n-1}]$.
