M 6575

Reg. No. : .....

Name : .....



**II Semester B.Sc. Degree (CCSS – Reg./Supple./Improv.) Examination, May 2014**
**COMPLEMENTARY COURSE IN MATHEMATICS**
**2C02 MAT : Differential and Integral Calculus**

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks :

   a) $\frac{d}{dx}\left(x^{1/n}\right) =$ _____

   b) $\frac{d}{dx}(\sinh^{-1}x) =$ _____

   c) $\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{\sin^2 x}\right) =$ _____

   d) Radius of curvature in parametric form is _____ **(Wt.1)**

Answer **any six** from the following (Weightage **1 each**).

2. Find the radius of curvature at the point (1, 1) on the curve $x^3 + y^3 = 2xy$.

3. If $y^{1/m} + y^{-1/m} = 2x$, prove that $(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0$.

4. Evaluate $\iint xy(x + y)\,dxdy$ over the area between $y = x^2$ and $y = x$.

5. Find the length of the arc of the parabola $x^2 = 4ay$ measured from the vertex to one extremity of the latus rectum.

6. If in the Cauchy's Mean Value Theorem, $f(x) = e^x$ and $F(x) = e^{-x}$, show that c is the arithmetic mean between a and b.

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7. Evaluate $\lim_{x\to 0}\left(\frac{1-\cos x}{x\sin x}\right)$.

8. Evaluate $\int_0^{\frac{\pi}{2}}\int_0^{2a\cos\theta} r^3\,dr d\theta$.

9. Find the area enclosed by $y = 3x^2 - x - 3$ and $y = -2x^2 + 4x + 7$.

10. Evaluate $\int_0^1\int_0^2\int_0^3 xyz\,dx\,dy\,dz$. **(6×1=6)**

Answer **any seven** from the following (Weightage **2 each**).

11. By changing the order of integration, evaluate $\int_0^\infty\int_0^\infty \frac{e^{-y}}{y}\,dy\,dx$.

12. Expand the polynomial $x^4 - 5x^3 + 5x^2 + x + 2$ in powers of $x - 2$.

13. Differentiate $y = x^{\tan x} + (\sin x)^{\cos x}$.

14. Show that for the curve $\frac{r}{\rho} = \sin\varphi\left(1+\frac{d\varphi}{d\theta}\right)$.

15. If $z = (x+y)\varphi\left(\frac{y}{x}\right)$, where $\varphi$ is any arbitrary function prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$.

16. Obtain a reduction formula for $\int x^m \sin nx\,dx$.

17. Find the whole area of the curve $xy^2 = a^2(a - x)$ and the y-axis.

18. Prove that the area of the region bounded by the curve $a^4y^2 = x^5(2a - x)$, is to that of the circle whose radius is a is 5 to 4.



19. Evaluate $\iint xy\,dxdy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$.

20. Find the volume common to the cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$. **(7×2=14)**

Answer **any three** from the following (Weightage **3 each**).

21. Find the evaluate of the astroid $x = a\cos^3\theta$, $y = a\sin^3\theta$.

22. Find the volume of the solid obtained by revolving one arc of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 + \cos\theta)$ about X-axis.

23. Find the area of the loop of the curve $x^4 + 3x^2y^2 + 2y^4 = a^2xy$.

24. Find the surface of the solid generated by the revolution of the lemniscates $r^2 = a^2\cos 2\theta$ about the initial line.

25. If $= x\log\frac{x-1}{x+1}$, prove that $\frac{d^ny}{dx^n} = (-1)^n(n-2)!\left[\frac{x-n}{(x+1)^n} - \frac{x+n}{(x+1)^n}\right]$. **(3×3=9)**