0105024

Reg. No. : ....

Name : ....

K19U 3320

I Semester B.Sc. Degree CBCSS(OBE) - Regular Examination, November - 2019
(2019 Admissions)
Core Course in Mathematics
1B01MAT : SET THEORY, DIFFERENTIAL CALCULUS AND NUMERICAL METHODS

Time : 3 Hours Max. Marks : 48

Part - A

Answer any 4 questions. Each Question carries 1 mark.

1. Find $gof\,(2)$ if $f : R \to R$ and $g : R \to R$ are given by $f(x)=3x-1$; $g(x)=x^2-2$.
2. Find the limit $\lim\limits_{x\to 0}\dfrac{\tan x}{x}$.
3. Find $\dfrac{\partial^2 z}{\partial x \partial y}$, if $z=\sin(2x-3y)$.
4. Find the degree of the homogenous equation $f(x,y)=\dfrac{\sqrt{x}}{x^2+y^2}$.
5. Is the relation R ={(1,1), (1,2), (2,2), (1,3), (2,3), (3,3), (2,1)} a partial order on {1,2,3}? Justify.

Part - B

Answer any 8 questions. Each question carries 2 marks.

6. Check if the function $f : R \to R$ given by $f(x)=\dfrac{3x+1}{2}$ is one-to-one and onto.
7. Define equivalence relation and check if R={(1,1), (1,2), (2,2), (3,3), (1,3)} on the set A = {1,2,3} is an equivalence relation or not.
8. Give an example of a function $f : R \to R$ which is one-to-one but not onto.
9. Write the inverse relation R^{-1}, if the relation R on the set A = {1,2,3,4,5} is given by R={(m,n):m'divide'n}
10. Find the symmetric closure and reflexive closure of the relation on the set A = (1,2,3,4) given by R={(1,1), (1,2), (1,3), (2,3)}.

P.T.O.

11. Find the limit $\lim_{x \to 0} \frac{x}{|x|}$, if exists. Justify your answer.

12. Find the points of discontinuity of the function $f(x) = \frac{x + 2x^2}{x^2 - 4x + 3}$, if any.

13. If a>0, $a \le f(x) \le a+x$, $a-x \le g(x) \le a$ and both the limits $\lim_{x \to 0} f(x), \lim_{x \to 0} g(x)$ exist, find $\lim_{x \to 0} \frac{f(x)}{g(x)}$.

14. If $z=u^2+v^2$ and $u=at^2$ $v=2at$, find $\frac{dz}{dt}$ using chain rule.

15. If $x^3+3x^2y+6xy^2+y^3=1$, find $\frac{dy}{dx}$.

16. Find a root of $xe^x-2=0$ using bisection method.

Part - C

Answer any 4 questions. Each question carries 4 marks.

17. Show that $f: R \to R$ defined by $f(x) = \frac{ax+b}{c}, a \neq 0, c \neq 0$ is one-to-one and onto. Find formula for f^{-1}.

18. Give an example of a function $f: R \to R$ whose limit does not exists at any point of R (with justifications)

19. Evaluate the following limits:

 i) $\lim_{x \to 3} \frac{x^3 - 27}{x^4 - 81}$

 ii) $\lim_{h \to 0} \frac{\sqrt{5h+4} - 2}{h}$

20. Examine the continuity of the function $f(x,y) = \begin{cases} \frac{3x-y}{2x+y} if (x,y) \neq (0,0) \\ 0, if (x,y) = (0,0) \end{cases}$ at the points (0,0) and (1,0).

21. If $u=e^x \cos(y)$, $v=e^x \sin(y)$ and $f(x, y)$ is any function of x and y, then show that

 i) $\frac{\partial f}{\partial x} = u\frac{\partial f}{\partial u} + v\frac{\partial f}{\partial v}$

 ii) $\frac{\partial f}{\partial y} = -v\frac{\partial f}{\partial u} + u\frac{\partial f}{\partial v}$

22. Show that if $y=f(x+at)+g(x-at)$ with f and g twice differentiable, then $\frac{\partial^2 y}{\partial t^2} = a^2\frac{\partial^2 y}{\partial x^2}$.

23. Derive the Newton-Raphson formula for finding the root of an equation.

Part - D

Answer any 2 questions. Each question carries 6 marks.

24. Let A = {1, 2, 3, ..., 9, 10}. The relation '~' on A x A is defined by $(a, b) \sim (c, d)$ if $ad = bc$. Check whether this is an equivalence relation. If so, find the equivalence classes [(1,1)], [(1,3)] and [(3,1)].

25. If $y = e^{a\sin^{-1}x}$, prove that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$. Hence find the value of y_n when x=0.

26. If $u = r^m$, where $r^2 = x^2 + y^2 + z^2$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = m(m+1)r^{m-2}$.

27. Find the point of intersection of the curve $y=x^3$ and the line $y=3x-1$ using regula-falsi method, starting with suitable initial approximations (correct to two decimal places).