Reg. No. : ....

Name : ....

K19U 3183

I Semester B.Sc. Degree (CBCSS-Supplementary) Examination, November - 2019

(2014-2016 Admissions)

CORE COURSE IN MATHEMATICS

1B01 MAT : DIFFERENTIAL CALCULUS

Time : 3 Hours Max. Marks : 48

SECTION - A

All the first **Four** questions are compulsory. They carry **1** mark each.

1. Evaluate $\lim_{x \to -1} \frac{x^3 + 4x^2 - 3}{x^2 + 5}$.

2. Find the value of $\cosh\frac{x}{3}$ if $\sinh\frac{x}{3} = \frac{4}{3}$.

3. Find the Cartesian coordinates corresponding to $(2, \frac{\pi}{2})$.

4. Transform the equation $x^2 + y^2 - 3z^2 = 0$ to spherical coordinates.

SECTION - B

Answer any **8** questions from among the questions **5 to 14**. These questions carry **2** marks each.

5. Evaluate $\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$.

P.T.O.

6. Suppose $\lim_{x\to c} f(x) = 5$ and $\lim_{x\to c} g(x) = -2$. Find $\lim_{x\to c} f(x)g(x)$ and $\lim_{x\to c} f(x) + 3g(x)$.

7. Prove that $\frac{d}{dx}\cosh^{-1}x = \frac{1}{\sqrt{x^2-1}}$.

8. Find the center and radius of the sphere whose equation is $x^2+y^2+z^2-\frac{2}{3}x-y-\frac{4}{3}z-\frac{22}{3}=0$.

9. The generators of a cylinder are parallel to the line $6x=-3y=2z$ and the guiding curve is given by $x^2+y^2=1, z=0$. Find its equation.

10. Verify Rolle's theorem for $f(x)=(x+2)^3(x-3)^4$ in (-2,3).

11. Using Maclaurin's series, expand log $(1+x)$.

12. Evaluate $\lim_{x\to 0^+} x\cot x$.

13. Find domain and the range of $w=\frac{1}{x^2+y^2+z^2}$.

14. Show that $f(x,y)=\begin{cases}\frac{2xy}{x^2+y^2}, & (x,y)\neq(0,0)\\ 0, & (x,y)=(0,0)\end{cases}$ is continuous at every point except at the origin.

SECTION - C

Answer any **Four** questions from among the questions **15 to 20**. These questions carry **4** marks each.

15. Show that $\lim_{x\to 1}(5x-3)=2$.

16. Verify Langrange's mean value theorem for $f(x)=(x-1)(x-2)(x-3)$ in (0.4) and find appropriate value for c.

17. Show that the radius of curvature at any point of the cycloid $x=a(\theta+\sin\theta), y=a(1-\cos\theta)$ is $4a\cos\frac{\theta}{2}$.

18. Find the asymptotes of the curve of $x^3+3x^2y-4y^3-x+y+3=0$.

19. Find the linearization $L(x,y,z)$ of $f(x,y,z)=x^2-xy+3\sin z$ at the point (2,1,0).

20. Verify Euler's theorem for $z=(x^2+xy+y^2)^{-1}$.

SECTION - D

Answer any **2** questions from among the questions **21 to 24**. These questions carry **6** marks each.

21. If $y=\sin(m\sin^{-1}x)$, prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(m^2-n^2)y_n=0$ also find $(y_n)_o$.

22. Show that the points (-4,3,6),(-5,2,2),(-7,6,6) and (-8,5,2) are concyclic.

23. Show that the evolute of the cycloid $x=a(t+\sin t), y=a(1-\cos t)$ is the curve $x=a(t-\sin t), y-2a=a(1+\cos t)$.

24. If $u=\tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$, $x\neq y$ show that.

i) $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=\sin 2u$

ii) $x^2\frac{\partial^2 u}{\partial x^2}+2xy\frac{\partial^2 u}{\partial x\partial y}+y^2\frac{\partial^2 u}{\partial y^2}=(1-4\sin^2 u)\sin 2u.$

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