K16U 2499

I Semester B.Sc. Degree (C.C.S.S. – Reg./Supple./Improv.)
Examination, November 2016
CORE COURSE IN MATHEMATICS
1B01 MAT : Differential Calculus
(2014 Admn. Onwards)

Time : 3 Hours

Total Marks : 48

SECTION – A

All the first 4 questions are **compulsory**. They carry **1** mark **each**.

1. State Sandwich Theorem.

2. The derivative of $\sin^{-1}(x^2)$ is _____

3. True or false : If $f'(a) = 0$ and $f''(a)$ is negative then $f(x)$ is maximum at $x = a$.

4. Find $\lim\limits_{(x,y)\to(3,4)} \sqrt{x^2 + y^2 - 1}$.

$(4\times1=4)$

SECTION – B

Answer **any 8** questions from among the questions **5** to **14**. They carry **2** marks each.

5. Find $\lim\limits_{x\to1}\dfrac{x-1}{\sqrt{x+3}-2}$.

6. Find the inverse of $y = \dfrac{1}{2}\,x + 1$, expressed as a function of x.

7. If $y = e^{ax}\sin bx$, prove that $y_2 - 2ay_1 + (a^2 + b^2)y = 0$.

8. Replace the polar equation $r^2 = 4r\cos\theta$ by equivalent Cartesian equation and identify the graph.

P.T.O.

9. Graph the set of points whose polar coordinates satisfy the inequalities

   $0 \leq r \leq 1$ and $\dfrac{\pi}{4} \leq \theta \leq \dfrac{3\pi}{4}$.

10. In the Mean Value Theorem $f(b) - f(a) = (b - a)\, f'(c)$. Determine 'c' lying between

    a and b if $f(x) = x(x - 1)\,(x - 2)$, $a = 0$ and $b = \dfrac{1}{2}$.

11. Find $\dfrac{ds}{dx}$ if $y = c\,\cosh(x/c)$.

12. Find $\displaystyle\lim_{x \to \infty} \dfrac{\ln x}{2\sqrt{x}}$.

13. Find the domain and range of the function $w = \dfrac{1}{xy}$.

14. Find $\dfrac{\partial f}{\partial y}$ if $f(x, y) = y \sin xy$.   **(8×2=16)**

SECTION – C

Answer **any 4** questions from **15** to **20**. They carry **4** marks **each**.

15. If $ax^2 + 2hxy + by^2 = 1$, find $\dfrac{d^2y}{dx^2}$.

16. Show that $f(x) = \dfrac{x^2 + x - 6}{x^2 - 4}$ has a continuous extension to $x = 2$ and find that extension.

17. Find 'ρ' at the origin for the curve $y^4 + x^3 + a(x^2 + y^2) - a^2 y = 0$.

18. Expand $e^{\sin x}$ by Maclaurin's series.

19. If $u = \dfrac{x^2 y^2}{x^2 + y^2}$, show that $\dfrac{x\partial^2 u}{\partial x \partial y} + \dfrac{y\partial^2 u}{\partial y^2} = \dfrac{\partial u}{\partial y}$.

20. Find a linearization of $f(x, y) = x^2 - xy + \dfrac{1}{2}y^2 + 3$ at the point $(3, 2)$.   **(4×4=16)**

SECTION – D

Answer **any 2** questions from **21** to **24**. They carry **6** marks **each**.

21. If $y = e^{a\sin^{-1}x}$, prove that $(1 - x^2)\, y_{n+2} - (2n + 1)\, x\, y_{n+1} - (n^2 + a^2)\, y_n = 0$.
    Hence find the value of y_n when $x = 0$.

22. a) Find an equation for the hyperbola with excentricity 3/2 and directrix $x = 2$.

    b) Find the directrix of the parabola $r = \dfrac{25}{10 + 10\cos\theta}$.

    c) Find the Cartesian coordinate of the point $(-3, 2\pi)$.

23. Show that the evolute of the cycloid $x = a\,(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$ is another equal cycloid.

24. If $\sin V = \dfrac{(x + 2y + 3z)}{\sqrt{x^8 + y^8 + z^8}}$, show that $x\dfrac{\partial v}{\partial x} + y\dfrac{\partial v}{\partial y} + z\dfrac{\partial v}{\partial z} + 3\tan V = 0$.

     (2×6=12)