Reg. No. : ....

Name : ....

K17U 2545

I Semester B.Sc. Degree (CBCSS – Reg./Supple./Improv.) Examination, November 2017
(2014 Admn. Onwards)
COMPLEMENTARY COURSE IN MATHEMATICS
1C01 MAT-ST : Mathematics for Statistics – I

Time : 3 Hours Max. Marks : 40

SECTION – A

All the first **4** questions are **compulsory**. They carry **1** mark **each**.

1. Find $\frac{d}{dx}(e^{3x} \cosh 4x)$.

2. State Taylor's theorem.

3. If $z = x^y$, find $\frac{\partial z}{\partial y}$.

4. Replace the polar equation, $r = -3 \sec \theta$ by an equivalent Cartesian equation.

(1×4=4)

SECTION – B

Answer **any 7** questions from among the questions **5** to **13**. These questions carry **2** marks **each**.

5. Find the derivative of $y = (\cos x)^{2x^2+3}$ with respect to x.

6. Find the second derivative of $f(x) = \frac{x-1}{x+2}$.

7. Find the n^{th} derivative of cos x cos 2x.

8. Find $\frac{dy}{dx}$, when $x = a(\cos t + \log \tan \frac{t}{2})$, $y = a \sin t$.

P.T.O.

9. Verify Rolles theorem for $f(x) = x + x^{-1}$ in the interval $\left[\frac{1}{2}, 2\right]$.

10. Find the intervals in which $f(x) = 3x^5 - 5x^3$ in increasing and decreasing.

11. Find $\lim_{(x,y)\to(0,0)} \frac{x^2y}{x^2+y^2}$.

12. Find $\frac{\partial^2 u}{\partial y \partial x}$ and $\frac{\partial^2 u}{\partial x \partial y}$ for $u = x \sin y + y \sin x$.

13. Find the coordinates of the centre of curvature at (c, c) of the curve $xy = c^2$.

(2×7=14)

SECTION – C

Answer **any 4** questions from among the questions **14** to **19**. These questions carry **3** marks **each**.

14. Obtain the expansion of log cosh x in powers of x by Maclaurin's theorem.

15. Evaluate $\lim_{x\to 0} \frac{e^x - 1 + \log(1-x)}{\sin^3 x}$.

16. Let $f : [a, b] \to \mathbb{R}$ be differentiable and $a \geq 0$. Using Cauchy mean value theorem, show that there exist $c_1, c_2 \in (a, b)$ such that $\frac{f'(c_1)}{a+b} = \frac{f'(c_2)}{2c_2}$.

17. If z is a function of x and y where $x = e^u + e^{-v}$, $y = e^{-u} - e^v$, prove that

$$\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}.$$

18. Find the radius of curvature of the curve $2y^2 = x^3$ at the point (2, 2).

19. Convert the coordinates $(2\sqrt{3}, 6, -4)$ from Cartesian to spherical. **(3×4=12)**

SECTION – D

Answer **any 2** questions from among the questions **20** to **23**. These questions carry **5** marks **each**.

20. If $y = \frac{\sinh^{-1} x}{\sqrt{1+x^2}}$, show that $(1 + x^2)\, y_{n+2} + (2n + 3)\, xy_{n+1} + (n + 1)^2 y_n = 0$.

21. Verify that the function $f(x) = x^3 - 2x$ satisfies the hypotheses of the Mean Value Theorem on the interval [–2, 2]. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

22. If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, prove that

$$\frac{d^2y}{dx^2} = \frac{abc + 2fgh - af^2 - bg^2 - ch^2}{(hx + by + f)^3}.$$

23. a) Translate the equation $\rho \sin \phi = 2$ into Cartesian equation.

b) Describe the graph $\theta = \pi/4$ in cylindrical coordinates. **(5×2=10)**

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