K17U 2544

Reg. No. : .....

Name : .....

I Semester B.Sc. Degree (CBCSS-Reg./Supple./Improv.) Examination, November 2017
(2014 Admn. Onwards)
COMPLEMENTARY COURSE IN MATHEMATICS
1C01 MAT-CH : Mathematics for Chemistry – I

Time : 3 Hours Max. Marks : 40

SECTION – A

All the **first 4** questions are **compulsory**. They carry **1** mark **each.**

1. Find the derivative of $\sinh^{-1}(x^5)$.

2. Find the value of $\frac{dy}{dx}$ if $x = 3t$, $y = t^2 - 4t + 1$.

3. Find $f_x(1, 3)$ for $f(x, y) = x^2 + y^2/4$.

4. In polar coordinates, what shape is described by $r = k$, where k is a constant ?

(1×4=4)

SECTION – B

Answer **any 7** questions from among the questions **5** to **13.** These questions carry **2** marks **each.**

5. If $y = a \cosh\frac{x}{a}$, prove that $a^2 y_2^2 = 1 + y_1^2$.

6. Find the n^{th} derivative of $e^{2x+4} + 6^{2x+4}$.

7. Expand cos x by Maclaurin's series.

P.T.O.

8. If $x^y \cdot y^x = 1$, find $\frac{dy}{dx}$.

9. Verify that $f(x) = x^3 - x^2 - 6x + 2$ satisfies the hypotheses of Rolle's theorem for the interval [0, 3], then find all c that satisfy the conclusion.

10. Determine $\lim_{x\to 1} \frac{1+\log x - x}{1-2x+x^2}$.

11. If $u = \log(e^x + e^y)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 1$.

12. If $u = \tan^{-1}\frac{y}{x}$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

13. Find the radius of curvature of $y = 3x^2 + 4x$ at (1, 7). **(2×7=14)**

SECTION – C

Answer **any 4** questions from among the questions **14** to **19**. These questions carry **3** marks **each**.

14. Expand log sin x in powers of $(x - 2)$.

15. Determine $\lim_{x\to 0}(\cos x)^{1/x^2}$.

16. Find the intervals in which the function f given by $f(x) = 4x^3 - 6x^2 - 72x + 30$ is (a) strictly increasing (b) strictly decreasing.

17. If $\sin v = \frac{x+2y+3z}{\sqrt{x^8+y^8+z^8}}$ show that $x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} + z\frac{\partial v}{\partial z} + 3\tan v = 0$.

18. For the cycloid $x = a(t + \sin t)$, $y = a(1 - \cos t)$, prove that $\rho = 4a\cos\frac{t}{2}$.

19. Write the point $(x, y, z) = \left(\sqrt{6}, -\sqrt{6}, -2\right)$ in cylindrical and spherical co-ordinates.

(3×4=12)

SECTION – D

Answer **any 2** questions from among the questions **20** to **23**. These questions carry **5** marks **each**.

20. If $y = \cos(m \sin^{-1}x)$ show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)\, y_n = 0$ and hence find $y_n(0)$.

21. Use Cauchy's mean value theorem to evaluate $\lim_{x\to 1}\left(\frac{\cos\frac{\pi x}{2}}{\log\frac{1}{x}}\right)$.

22. Show that the evolute of the ellipse $x = a\cos\theta$, $y = b\sin\theta$ is $(ax)^{2/3} + (by)^{2/3} = (a^2 - b^2)^{2/3}$.

23. a) Translate the equation $r = \csc\theta$ into Cartesian and spherical equations.

    b) Replace the polar equation $r^2 = 4r\sin\theta$ by an equivalent Cartesian equation. Describe the graph of the equation. **(5×2=10)**