K17U 2543

I Semester B.Sc. Degree (CBCSS - Reg./Supple./Improv.) Examination, November 2017

COMPLEMENTARY COURSE IN MATHEMATICS 1C01 MAT-PH: Mathematics for Physics and Electronics - I (2014 Admn. Onwards)

Time: 3 Hours

Max. Marks: 40

SECTION - A

All the first 4 questions are compulsory. They carry 1 mark each.

1. Find the derivative of log (cosh x).

2. Find
$$\frac{dy}{dx}$$
 if $x = 2t + 3$, $y = t^2 - 1$.

3. Find
$$\frac{\partial u}{\partial x}$$
 of $u = e^{xyz}$.

4. Convert the coordinates (3, $\pi/3$, -4) from cylindrical to Cartesian.

 $(1 \times 4 = 4)$

SECTION - B

Answer any 7 questions from among the questions 5 to 13. These questions carry 2 marks each.

5. If
$$x = a (\cos \theta + \theta \sin \theta)$$
, $y = a (\sin \theta - \theta \cos \theta)$, find $\frac{d^2y}{dx^2}$.

6. Find the nth derivative of
$$y = \frac{x^2}{(x+2)(2x+3)}$$
.

- 7. If $I_n = \frac{d^n}{dx^n} (x^n \log x)$, prove that $I_n = nI_{n-1} + (n-1)!$
- 8. Expand $\log (1 + x)$ by Maclaurin's theorem.
- 9. Let f(x) = (x a)(x b)(x c), a < b < c, show that f'(x) = 0 has two roots one belonging to (a, b) and other belonging to (b, c).
- 10. Determine $\lim_{x \to \frac{\pi}{2}} \frac{\tan 3x}{\tan x}$
- 11. If $u = x^2 \tan^{-1} \frac{y}{x} y^2 \tan^{-1} \frac{x}{y}$; $xy \neq 0$, prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 y^2}{x^2 + y^2}$.
- 12. If $u = \tan^{-1} \frac{x^3 + y^3}{x y}$, $x \neq y$ show that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$.
- 13. Find $\frac{dz}{dt}$ when $z = xy^2 + x^2y$, $x = at^2$, y = 2at. (2x7=14)

SECTION-C

Answer any 4 questions from among the questions 14 to 19. These questions carry 3 marks each.

- 14. Expand $2x^3 + 7x^2 + x 6$ in powers of (x 2).
- 15. Separate the intervals in which $f(x) = 2x^3 15x^2 + 36x + 1$ is increasing or decreasing.
- 16. Find $\lim_{x\to 0} \left(\frac{\tan x}{x}\right)^{\frac{1}{x}}$.
- 17. If $u = \log (x^3 + y^3 + z^3 3xyz)$, show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$.

- 18. Show that the radius of curvature at any point of the catenary $y = c \cosh \frac{x}{c}$ varies as the square of the ordinate.
- 19. Find the Cartesian equation equivalent to the polar equation, $r = \frac{4}{2\cos\theta \sin\theta}$ and draw its graph. (3x4=12)

SECTION - D

Answer any 2 questions from among the questions 20 to 23. These questions carry 5 marks each.

- 20. If $y = \sin(m \sin^{-1} x)$, show that $(1 x^2) \frac{d^2y}{dx^2} x \frac{dy}{dx} + m^2y = 0$. Hence expand $\sin m\theta$ in powers of θ .
- 21. Show that $\frac{2}{\pi} < \frac{\sin x}{x} < 1, 0 < x < \frac{\pi}{2}$.
- 22. Find the evolute of the parabola, $y^2 = 4ax$.
- 23. Translate the equation $\rho = 5\cos\phi$ into Cartesian and cylindrical equations.

 $(5 \times 2 = 10)$