K15U 0473

Reg. No. : ....

Name : ....

I Semester B.Sc. Degree (CCSS – Supple./Improv.) Examination, November 2015
(2013 and Earlier Admn.)
COMPLEMENTARY COURSE IN MATHEMATICS
1C01 MAT : Algebra and Geometry

Time : 3 Hours Max. Weightage : 30

1. Fill in the blanks.
   a) Number of identity elements in a group is ____________
   b) Dimension of R^3 is ____________
   c) The condition for unique solution for the system of homogeneous system of equations $AX = 0$ is ____________
   d) If A is a square matrix of order n, then the roots of the equation $|A - \lambda I| = 0$ are known as ____________ **(Weightage 1)**

Answer **any six** from the following. (Weightage **1 each**)

2. Define a field.
3. Give an example for a vector space.
4. Define linear transformation between two vector spaces.
5. Examine whether the transformation $T : R^3 \rightarrow R$ defined by $T(x, y, z) = xy + x + z$ is linear or not. Justify your answer.
6. State Cayley Hamilton theorem.
7. If $\lambda = 1$ is an eigen value of $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$, find the other two eigen values of A without using its characteristic equation.

P.T.O.

8. Prove that eigen values of a skew-Hermitian matrix are purely imaginary or zero.

9. Sketch the graph $-3 \le r \le 2;\ \theta = \frac{\pi}{4}$.

10. Obtain the relation between spherical coordinates and Cartesian co-ordinates in space. **(Weightage 6×1=6)**

Answer **any seven** from the following. (Weightage **2 each**)

11. Let S be the set of all real numbers except – 1. Define $*$ on S by $a * b = a + b + ab$. Show that $\langle S, * \rangle$ is a group.

12. Define linearly dependence and independence of vectors. Examine whether the Vectors (2, 2, 1), (1, –1, 1), (1, 0 , 1) are linearly independent.

13. Let $T : R^3 \to R^3$ be a linear transformation defined by $T(x, y, z) = (0, x + y, x + y + z)$. Find the matrix representation of T with respect to the ordered basis $X = \{(1, 0, 1), (1, 1, 0), (0, 1, 1)\}$ in R^3 and $Y = \{(1, 0, 0)\ (0, 1, 0), (0, 0, 1)\}$ in R^3.

14. Let $T : R^3 \to R^2$ be a linear transformation defined by $T(x, y, z) = (y + z, y - z)$. Find the matrix of the linear transformation T with respect to the ordered basis $X = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ in R^3 and $Y = \{(1, 0), (0, 1)\}$ in R^2.

15. Reduce the matrix $A = \begin{bmatrix} 4 & 3 & 0 & -2 \\ 3 & 4 & -1 & -3 \\ 7 & 7 & -1 & -5 \end{bmatrix}$ into its normal form and hence find its rank.

16. Show that equations $x + 2y - z = 3$, $3x - y + 2z = 1$, $2x - 2y + 3z = 2$, $x - y + z = -1$ are consistent and solve them.

17. For what values of λ and μ do the system of equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have (a) no solution (b) unique solution (c) more than one solution ?

18. Find all the non-trivial solutions of $2x - y + 3z = 0$, $3x + 2y + z = 0$, $x - 4y + 5z = 0$.

19. Find the equivalent Cartesian equation of the polar equation $r = \frac{4}{2\cos\theta - \sin\theta}$.

20. Find a spherical coordinate equation for the cone $z = \sqrt{x^2 + y^2}$. **(Weightage 7×2=14)**

Answer **any three** from the following. (Weightage **3 each**)

21. Determine whether the following set of vectors {u, v, w} form a basis in R^3, where

 i) u = (2, 2, 0), v = (3, 0, 2), w= (2, – 2, 2)

 ii) u = (0, 1, –1), v = (–1, 0, –1), w = (3, 1, 3).

22. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$.

23. Verify Cayley Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 0 & -2 \\ 2 & 2 & 4 \\ 0 & 0 & 2 \end{bmatrix}$ and hence find A^4.

24. Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ and hence find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$.

25. Describe the rectangular, cylindrical and spherical coordinate representation of points in space. Represent the rectangular equation $x^2 + y^2 + \left(z - \frac{1}{2}\right)^2 = \frac{1}{4}$ into other two coordinate systems. **(Weightage 3×3=9)**