M 7773

Reg. No. : ....

Name : ....

I Semester B.Sc. Degree (CCSS – Supple./Improve.)
Examination, November 2014
COMPLEMENTARY COURSE IN MATHEMATICS
1C 01 MAT : Algebra and Geometry
(2013 and Earlier Admn.)

Time: 3 Hours Max. Weightage : 30

1. Fill in the blanks :

   a) ________________ is an example of a nonabelian group.

   b) ________________ is an example of a two dimensional vector space.

   c) ________________ is an example of a field.

   d) ________________ is a subspace of $\mathbb{R}^3$. **(Weightage – 1)**

Answer **any six** from the following (Weightage **1 each**) :

2. Find the span of $\{(1, 1), (2, 2)\}$ in $\mathbb{R}^2$.

3. Prove or disprove that $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x_1, x_2) = (2x_1, 3x_2)$ is a linear transformation.

4. Check whether the set of all $f \in \mathcal{C}[0, 1]$ such that $f\left(\frac{3}{4}\right) = 1$ is a subspace of $\mathcal{C}[0, 1]$.

5. Show that in a vector space V any set of vectors containing the zero vector is linearly dependent.

6. Let $T : U \to V$ be a linear map. Then prove that $T(-u) = -T(u)$.

7. Can we produce any number of basis in a vector space. Why ?

8. Define eigen value of a matrix.

9. Can polar coordinates have negative values ? Explain.

10. Write equations relating rectangular (x, y, z) and cylindrical (r, θ, z) co-ordinates.

11. Find an equation for the cylinder $x^2 + (y - 3)^2 = 9$ in cylindrical co-ordinates.

(Weightage 6×1=6)

Answer **any seven** from the following (weightage **2 each**) :

12. Let S be a nonempty subset of a vector space V. Then prove that [S], the span of S, is a subspace of V.

13. Let U_1 and U_2 be two subspaces of a vector spce V. Then prove that $U_1 \cap U_2$ is also a subspace of V.

14. Prove that in an n-dimensional vector space V, any set of n linearly independent vector is a basis.

15. Prove that a linear transformation on a 1-dimensional vector space is nothing but multiplication by a fixed scalar.

16. Determine whether there exists a linear map $T : V_2 \to V_2$ such that $T(2, 1) = (2,1)$ and $T(1, 2) = (4, 2)$. If it exists write the general formula otherwise give reasons.

17. Find the rank of the matrix :

$$\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$$

18. Using Cayley Hamilton theorem, show that $A^3 - 6A^2 + 11A - 6I = 0$ where

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$$

19. Investigate the values of λ and μ so that the equations $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$ have no solution.

20. Solve the system of equations :

$2x - y + z = 7$, $3x + y - 5z = 13$, $x + y + z = 5$.

21. Show that if $\lambda \neq -5$, the system of equations :

$3x - y + 4z = 3$, $x + 2y - 3z = -2$, $6x + 5y + \lambda z = -3$ have a unique solution.

22. Show that the transpose A^T has the same eigen values of A. **(7×2=14)**

Answer **any three** from the following (Weightage **3 each**) :

23. Find the eigen values and the corresponding eigen vectors of the matrix :

$$\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$$

24. Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$ and find its inverse.

25. 1) Convert the polar equation $r = 8\sin\theta$ into Cartesian equation.

    2) Convert the Cartesian equation $y^2 = 4x$ into polar equation.

26. Translate $x^2 + y^2 + (z - \frac{1}{2})^2 = \frac{1}{4}$ into cylindrical and spherical system.

(Weightage : 3×3=9)