K20U 0235

Reg. No. : ....

Name : ....

II Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, April 2020
(2016 Admission Onwards)
BHM 204 : THEORY OF NUMBERS AND EQUATIONS

Time : 3 Hours Max. Marks : 60

Any 4 out of 5 questions. **Each** question carries **1** mark.

1. If a|b and a|c then show that a|(br + cs) for arbitrary integers r and s.
2. Check whether the Diophantine equation $14x + 35y = 93$ is solvable or not.
3. For any positive integers a and b, prove that lcm (a, b) = ab if and only if gcd(a, b) = 1.
4. State Wilson's Theorem.
5. Remove the fractional coefficients from the equation $x^3 - \frac{1}{4}x^2 + \frac{1}{3}x - 1 = 0$.

(4×1=4)

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. If gcd(a, b) = 1 and gcd(a, c) = 1 then show that gcd(a, bc) = 1.
7. Prove that for any integer a, one of the integers a, a + 2, a + 4 is divisible by 3.
8. Use Euclidean algorithm to obtain integers x and y satisfying gcd(112, 256) = 112x + 256y.
9. Show that there are infinitely many composite numbers of the form n! + 1.
10. If gcd(a, b) = 1 then show that gcd(a + b, a – b) = 1 or 2.
11. If $a \equiv b \pmod{n}$ and m|n then show that $a \equiv b \pmod{m}$.

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12. If $x_1, x_2, \ldots, x_n$ are the roots of the equation $(a_1 - x)(a_2 - x)\ldots(a_n - x) + k$, then show that $a_1, a_2, \ldots, a_n$ are the roots of the equation $(x_1 - x)(x_2 - x)\ldots(x_n - x) - k = 0$.

13. Frame an equation with rational coefficients, one of whose root is $\sqrt{5} + \sqrt{2}$.

14. Determine completely the nature of the roots of the equation $x^5 - 6x^2 - 4x + 5 = 0$. **(6×2=12)**

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. State and prove Euclid's theorem.

16. If $a \equiv b(\text{mod } n)$, then show that $a + c \equiv b + c(\text{mod } n)$ and $ac \equiv bc(\text{mod } n)$.

17. Find the solutions of the system of congruences $7x + 3y \equiv 10(\text{mod } 16)$, $2x + 5y \equiv 9(\text{mod } 16)$.

18. Using Chinese Remainder theorem solve the linear congruences $x \equiv 2(\text{mod } 3)$, $x \equiv 3(\text{mod } 5)$ and $x \equiv 2(\text{mod } 7)$.

19. Find the remainder when 2(26!) is divided by 29.

20. For any integer $k \neq 0$, prove that $\gcd(ka, kb) = |k|\gcd(a, b)$.

21. Find all prime numbers that divide 50!.

22. Let a and b be integers, not both zero. Show that for a positive integer d, $\gcd(a, b) = d$ if and only if
    a) $d|a$ and $d|b$
    b) whenever $c|a$ and $c|b$ implies that $c|d$.

23. A farmer purchased 100 head of livestock for a total cost of Rs. 4,000. Prices were as follows : calves, Rs. 120 each; lambs, Rs. 50 each; piglets, Rs. 25 each. If the farmer obtained at least one animal of each type, how many of each did he buy ?

24. Solve the equation $x^6 - 4x^5 - 11x^4 + 40x^3 + 11x^2 - 4x - 1 = 0$ given that one root is $\sqrt{2} - \sqrt{3}$.

25. Solve the equation $x^3 - 9x^2 + 108 = 0$ using Cardan's method.

26. Solve the equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ given that two of its roots are equal in magnitude and opposite in sign. **(8×4=32)**

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. a) If $p \geq 5$ is a prime number, show that $p^2 + 2$ is composite.
    b) Prove that $3\sqrt{2}$ is an irrational number.

28. The basket-of-eggs problem is often phrased in the following form: One egg remains when the eggs are removed from the basket 2, 3, 4, 5, or 6 at a time; but, no eggs remain if they are removed 7 at a time. Find the smallest number of eggs that could have been in the basket.

29. If $\alpha, \beta, \gamma, \delta$ be the roots of the bi quadratic $x^4 + px^3 + qx^2 + rx + s = 0$. Find
    a) $\Sigma\alpha^2$
    b) $\Sigma\alpha^2\beta^2$
    c) $\Sigma\alpha^2\beta\gamma$
    d) $\Sigma\alpha^3\beta$
    e) $\Sigma\alpha^4$

30. Solve the equation $6x^5 - x^4 - 43x^3 + 43x^2 + x - 6 = 0$. **(2×6=12)**