K17U 1388

Reg. No. : ....

Name : ....

VI Semester B.Sc. Hon's (Mathematics) Degree (Regular/Supple./Improv.) Examination, May 2017
(2013 Admission)
BHM 603 : TOPOLOGY

Time : 3 Hours Max. Marks : 80

Answer **all** the **ten** questions : **(10×1=10)**

1. Give examples of two subsets A and B of $\mathbb{R}$ such that $\text{int}(A) \cup \text{int}(B) \neq \text{int}(A \cup B)$.
2. Given a metric space (X, d) and $A \subseteq X$ what is the induced metric on A ?
3. If (X, d) is the discrete metric space, what is B(x, 1) ?
4. What is the closure of $\mathbb{Z}$ in $\mathbb{R}$?
5. Give an example of a metric space which is not complete.
6. Define the semi open interval topology on $\mathbb{R}$.
7. Give an example of a dense subset of $\mathbb{R}$ with usual topology.
8. If X and Y are non empty sets then give a topology on X that makes every function $f : X \to Y$ continuous.
9. Define a path in a topological space X.
10. Given a non empty set X, which is the weakest topology on X that makes X into a T_1 space ?

Answer **any 10** short answer questions out of 14 : **(10×3=30)**

11. Show that (a, b) is an open ball in $\mathbb{R}$ with standard metric. Is $\mathbb{R}$ itself an open ball ? Justify.

P.T.O.

12. If A and B are two dense subsets of a metric space,

    i) Is $A \cup B$ dense ?

    ii) Is $A \cap B$ dense ?

    Justify.

13. Define a sub-base for a topology τ on X. Give a sub-base for $\mathbb{R}$ with usual topology.

14. If $\{U_n : n \in \mathbb{N}\}$ is collection of open sets in $\mathbb{R}$, is it true that $\bigcap_{n \in \mathbb{N}} U_n$ is open ? Justify.

15. Let $\{(X_i, \tau_i)\ i = 1, 2, \ldots, n\}$ be a collection of topological spaces and (X, τ) their topological product. Prove that each projection function π_i is continuous.

16. An infinite set with cofinite topology is T_2. Prove or disprove.

17. Prove that a subset of a topological space is open if and only if it is a neighborhood of each of its points.

18. Prove that $\mathbb{R}$ with semi open interval topology is separable but not second countable.

19. Prove that continuous bijection from a compact space on to a Hausdorff space is a homeomorphism.

20. Prove that a space X is disconnected if and only if there exists two non empty closed sets A and B of X with $X = A \cup B$ and $A \cap B = \phi$.

21. Show that if A is a bounded subset of R then its supremum and infimum are limit points of A.

22. Show that $\left\{ B\left(x, \frac{1}{n}\right) : x \in \mathbb{R}, n \in \mathbb{N} \right\}$ forms a base for the usual topology on $\mathbb{R}$.

23. Prove that a closed subspace of a compact space is compact.

24. Prove that every infinite subset A of a compact space X has atleast one accumulation point in X.

Answer **any 6** short answer questions out of **9** : **(6×5=30)**

25. Let $X = \mathbb{R} \times \mathbb{R}$. Define $d : X \times X \to \mathbb{R}$ by $d((x_1, x_2), (y_1, y_2)) = |x_1 - y_1| + |x_2 - y_2|$. Show that d is a metric on X.

26. Prove that the limit of a sequence in a metric space is unique.

27. Show that any two open intervals in $\mathbb{R}$ are homeomorphic.

28. If X is a non empty set and S is any collection of subsets of X then show that S is a subbase for some topology on X.

29. Define hereditary topological property. Show that complete regularity is a hereditary property.

30. For a subset A of a topological space X show that $\overline{A} = \text{int}(A) \cup \text{boundary}(A)$.

31. All metric spaces are T_4. Prove or disprove.

32. Let X be an uncountable set with cofinite topology on it. Prove that X is separable but not first countable at any point.

33. When do you say that two subsets A and B of a space X are mutually separated. Let C be a collection of connected subsets of a space X such that no two members are mutually separated. Show that $\bigcup_{C \in \mathcal{C}} C$ is also connected.

Answer **any one** essay questions out of 2 : **(1×10=10)**

34. Prove that in a topological space X the following statements are equivalent :

    i) X is locally connected.

    ii) Components of open subsets of X are open.

    iii) X has a base consisting of connected subsets.

    iv) For every $x \in X$ and every neighborhood N of x there exists an open connected neighborhood M of x such that $M \subseteq N$.

35. Define *Lebesgue number.* Prove that in a compact metric space every open cover has a Lebesgue number. Using this result show that every continuous function on a compact metric space is uniformly continuous.
