K16U 1278

Reg. No. : ....

Name : ....

VI Semester B.Sc. (Hon's.) (Mathematics) Degree (Regular) Examination, May 2016

BHM 603 : TOPOLOGY

Time : 3 Hours Max. Marks : 80

Answer **all** the **ten** questions : **(10×1=10)**

1. Define *Discrete metric.*
2. Given a non-empty set X. What is the cofinite topology on X ?
3. Give an example of a hereditary topological property.
4. Define *Homeomorphism.*
5. What is the closure of $\mathbb{Q}$ in $\mathbb{R}$?
6. Give an example of a complete metric space.
7. Given a metric space (X, d) explain the term B_d [x, r] where $x \in X$ and $r > 0$.
8. If (X, τ) is a topological space and $\mathcal{C}$ is the closure operator what are the fixed points of $\mathcal{C}$.
9. When does equality hold in the triangle equality for the absolute value metric in $\mathbb{R}$, that is d (x, z) = d (x, y) + d (y, z) ?
10. For a connected topological space X which are the clopen subsets.

Answer **any 10** short answer questions out of 14 : **(10×3=30)**

11. If in a metric space B (x, r) = B (x, s), does it mean x = y and r = s. Justify.
12. Let A be any finite set in a metric space (X, d). Show that X\A is open.

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13. Given two sets A and B in metric space (X, d), $\overline{A \cap B} = \overline{A} \cap \overline{B}$. Prove or disprove.

14. Is every Cauchy sequence in a metric space convergent ? Justify your answer.

15. Suppose X is an infinite set with cofinite topology. Show that X is T_1.

16. Suppose (X, τ) is a space and $Y \in \tau$. Prove that $B \subset Y$ is open in Y if and only if it is open in X.

17. Prove that a discrete space is second countable if and only if the underlying set is countable.

18. Show that a subset A of topological space X is dense in X if and only if for every non-empty open set B of X, $A \cap B \neq \phi$.

19. Let X be a space $A \subseteq X$. Prove that Int (A) is the union of all open sets contained in A.

20. Give an example of a function $f : \mathbb{R} \to \mathbb{R}$ which is not continuous at any point.

21. If (X, τ_1), (Y, τ_2) and (Z, τ_3) are topological spaces and $f : X \to Y$ and $g : Y \to Z$ are continuous functions, show that $gof : X \to Z$ is also continuous.

22. Prove that every second countable space is separable.

23. Prove that a closed subspace of a compact space is compact.

24. Is the continuous image of a connected space connected ? Justify your claim.

Answer **any 6** short answer questions out of **9** : **(6×5=30)**

25. Let (X, d) be a metric space. Define $\delta(x, y) = \dfrac{d(x, y)}{1 + d(x, y)}$ for all $x, y \in X$. Show that δ is a metric on X.

26. In a metric space (X, d) show that the collection of all open sets is closed under finite intersections and arbitrary unions.

27. Does there exist an infinite subset of $\mathbb{R}$ with out a cluster point in $\mathbb{R}$? Substantiate your claim.

28. Show that Metrisability is a hereditary property.

29. State and prove a necessary sufficient condition for a family $\mathcal{B}$ of subsets of X covering X to be a base for a topology on X.

30. For a subset A of a topological space X show that $\overline{A} = A \cup A'$ where A′ is the derived set of A.

31. Let $\{(X_i, \tau_i)\, i = 1, 2, ..., n\}$ be a collection of topological spaces and (X, τ) their topological product. Prove that each projection function π_i is continuous. If Z is any space then show that a function $f : Z \to X$ is continuous if and only if each $\pi_i of : Z \to X_i$ is continuous.

32. Suppose X is a Haursdorff space and $\{x_n\}$ is a sequence in x. If $x_n \to x$ and $x_n \to y$ show that x = y. Give an example of non-Haursdorff space where this fails.

33. Show that every quotient space of a locally connected space is locally connected.

Answer **any one** essay questions out of **2** : **(1×10=10)**

34. Prove that a space X is normal if and only if whenever $A \subseteq X$ is closed and $f : A \to [-1, 1]$ is continuous, there exists a continuous function $F : X \to [-1, 1]$ such that $F(x) = f(x)$ for every $x \in A$.

35. i) Identify the connected subsets of $\mathbb{R}$. Justify your answer.

ii) If $A = \left\{\left(x, \sin\left(\frac{1}{x}\right)\right) : 0 \le x < 1\right\} \subseteq \mathbb{R}^2$ and let $X = \overline{A}$ where the closure is with respect to usual topology on $\mathbb{R}^2$. Prove that X is connected.

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