K20U 0214

Reg. No. : ....

Name : ....

VI Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, April 2020
BHM 602 : TOPOLOGY
(2016 Admissions Onwards)

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any four** questions out of the **five** questions. **Each** question carries 1 mark. **(4×1=4)**

1. If X = {a, b, c, d} and $\Im = \{\phi, \{a\}, \{a, b\}, \{a, b, d\}, \{a, b, c, d\}\}$, test whether $\Im$ is a topology on X.
2. If X = {a, b} and $\Im = \{\phi, \{a\}, \{a, b\}\}$, is $(X, \Im)$ metrizable. Justify your answer.
3. Is it true that every topological space has at least one basis. Justify your answer.
4. Is it true that [0, 1] homeomorphic to (0, 1) ? Justify your answer.
5. Establish with the help of an example that not every T_0 space is T_1.

SECTION – B

Answer **any six** questions out of the **nine** questions. **Each** question carries 2 marks. **(6×2=12)**

1. Prove that in the discrete topological space $(X, \Im)$, a sequence is convergent if and only if it is eventually constant.
2. Prove that the semi open interval topology on the set of real numbers R is stronger than the usual topology on R.
3. Prove that the co finite topology on any infinite set X has countable dense subsets.
4. If $\Im_1$ and $\Im_2$ are two topologies on a set X, is $\Im_1 \cup \Im_2$ a topology on X ? Justify your answer.
5. If $(X, \Im)$ is a topological space and $Y \in \Im$, prove that a subset G of Y is open in the subspace topology $(Y, \Im/_Y)$ if and only if G is open in $(X, \Im)$.

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6. Prove that second countability is a hereditary property.
7. Is there any topological space in which compact subsets are not closed ? Justify.
8. Is it true that the set of rational numbers Q is connected in the subspace topology on Q induced by the usual topology on the set of real numbers R ? Justify your answer.
9. Is the cofinite topology on an infinite set X is Hausdorff ? Justify. What happens if X is finite ? Justify.

SECTION – C

Answer **any eight** questions out of the **twelve** questions. **Each** question carries **4** marks. **(8×4=32)**

1. Prove that the set of rational numbers is dense in the set of real numbers in the usual topology.
2. Let X be a non empty set and A and B are two non empty distinct and proper subsets of X. Prove that $\Im = \{\phi, A, B, X\}$ is a topology on X if and only if exactly one of the following holds.
   1) A is a proper subset of B
   2) B is a proper subset of A
   3) A = X – B.
3. Let $(X, \Im_1)$ and $(Y, \Im_2)$ be two topological spaces and every function from $X \to Y$ is continuous from $(X, \Im_1)$ to $(X, \Im_2)$. Prove that either $\Im_1$ is discrete or $\Im_2$ is indiscrete.
4. Let $(X, \Im)$ be a topological space. Prove that a subset A of X is a dense subset of X if and only if for every non empty open subset B of X, $A \cap B \neq \phi$.
5. Define a quotient map and prove that every closed surjective map is a quotient map.
6. Prove that the continuous image of a compact set is compact.
7. Prove that a space is connected if and only if it cannot be written as the disjoint union of two non empty open subsets.
8. Let $(X, \Im)$ be a topological space and C is a subset X with $(C, \Im/C)$ is connected. If $C \subset A \cup B$, where A and B are mutually separated subsets of X, Prove that either $C \subset A$ or $C \subset B$.
9. Prove that every second countable space is first countable. Is the converse true ? Justify.

10. Prove that a space $(X, \Im)$ is a T_1 space if and only if every $\{x\}$, $x \in X$, is closed.
11. Prove that in a Hausdorff space, the limits of sequences are unique.
12. Define a completely regular space and prove that every completely regular space is regular.

SECTION – D

Answer **any two** questions from the four questions. **Each** question carries **6** marks. **(2×6=12)**

1. a) Let X be a set and D a family of subsets of X. Prove that there is a unique topology $\Im$ on X, such that it is the smallest topology on X containing D.
   b) Define a sub base for a topology $\Im$ on a set X. Give a countable sub base for the usual topology on R.
2. a) Let $(X, \Im)$ be a topological space and $A \subset X$. Prove that A is a compact subset of X if and only if $(A, \Im/A)$ is a compact topological space.
   b) Is it true that closure of a compact subset is always compact ? Justify your answer.
3. a) Prove that the closure of a connected set is always connected. Is the interior of a connected set is always connected ? Justify your answer.
   b) Is there any continuous bijection from [a, b] to (a, b) in the usual topology ? Justify your answer.
4. a) If $(X, \Im)$ is a Hausdorff space and $K = \{(x, x) : x \in X\} \subset X \times X$, prove that K is a closed subset relative to the product topology on $X \times X$.
   b) Let $(X, \Im)$ be a regular space. Prove that for any $x \in X$ and any open set G containing x, there is an open set H such that $x \in H \subset \overline{H} \subset G$.

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