K19U 0777

Reg. No. : ....

Name : ....

VI Semester B.Sc. Hon's (Mathematics) Degree (Supplementary) Examination, April 2019
(2013-'15 Admissions)
BHM 603 : TOPOLOGY

Time : 3 Hours Max. Marks : 80

Answer **all** the **ten** questions : **(10×1=10)**

1. Describe the discrete metric and taxicab metric on $\mathbb{R}^n$.
2. State Minkowski inequality.
3. Define continuity of a sequence.
4. Find the closure of a set $\mathbb{Q} \subset \mathbb{R}$ under usual-topology.
5. State whether true or false and justify your answer. $\mathbb{R}$ under usual topology is hausdorff.
6. Define base for a topology.
7. Find the boundary of A = {a}, where X = {a, b} with discrete topology.
8. State Lebesgue covering lemma.
9. Define local base at a point.
10. What is T_1 axiom ?

Answer **any 10** short answer questions out of 14. **(10×3=30)**

11. Show that limit of a sequence in a metric space is unique.
12. State Holder's inequality.
13. Prove that $\mathbb{R}$ is complete under usual metric.

P.T.O.

14. State Weierstrass approximation theorem.

15. Consider real line under cofinite topology. Find the closure of $\mathbb{Z}$.

16. Prove that int A $\cup$ int B $\subseteq$ int (A $\cup$ B).

17. Let A be a subset of a topological space (X, τ). Prove that A is closed if and only if $\bar{A} = A$.

18. Let τ_1, τ_2 be two topologies for a set having bases $\mathbb{B}_1$ and $\mathbb{B}_2$ respectively. Then prove that τ_1 is weaker than τ_2 if and only if every member of $\mathbb{B}_1$ can be expressed as a union of some members of $\mathbb{B}_2$.

19. Prove that every open surjective map is a quotient map.

20. Prove that if f is continuous at x_0 then the inverse image of every neighbourhood of $f(x_0)$ in Y is a neighbourhood of x_0 in X.

21. Show that every second countable space is first countable.

22. Prove the closed subspace of a compact space is compact.

23. Prove or disprove : Every completely regular space is regular.

24. Show that a compact subset in a Hausdorff space is closed.

Answer **any 6** short answer questions out of 9. **(6×5=30)**

25. Let X and Y be metric space. Let X × Y enclosed with the product metric. Show that sequence $(x_n, y_n) \in X \times Y$ converges to $(x, y) \in X \times Y$ if and only if $x_n \to x$ in X and $y_n \to y$ in Y.

26. State and prove Bolzano-Weierstrass theorem.

27. Let A be a subset of a space X. Prove that A is dense in X if and only if int $(X - A) = \varnothing$.

28. Show that metrizability is a hereditary property.

29. Prove that a set is closed if and only if it contains its boundary and it is open if and only if it is disjoint from its boundary.

30. For any three spaces, X, Y, Z, prove that X × (Y × Z) id homeomorphic to (X × Y) × Z.

31. Prove that every continuous real valued function on a compact space is bounded and attains its extrema.

32. Prove that all metric space are T_4.

33. Prove that a Hausdorff space limits of sequence are unique.

Answer **any one** essay question out of 2. **(1×10=10)**

34. Prove that every closed bounded interval is compact.

35. Let $\{(X_i, \tau_i) ; i = 1, 2, ..., n\}$ be a collection of topological spaces and (X, τ) be their product. Prove

a) Each projection π_i is continuous.

b) If Z is any space then the function $f : Z \to X$ is continuous if and only if $Z \mapsto X_i$ is continuous for all i = 1, 2, ..., n.