K19U 0762

Reg. No. : ....

Name : ....

VI Semester B.Sc. Hon's (Mathematics) Degree (Reg.) Examination, April 2019
(2016 Admission)
BHM 602 : TOPOLOGY

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any four** questions out of the five questions. **Each** question carries **1** mark. **(4×1=4)**

1. If $X = \{a, b, c, d, e\}$ and $\Im = \{\phi, \{a\}, \{a, b\}, \{a, b, d\}, \{a, b, c, d\}, X\}$, test whether $\Im$ is a topology on X.
2. If $X = \{a, b, c\}$ and $\Im = \{\phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$, is $(X, \Im)$ metrizable. Justify your answer.
3. If $(X, \Im)$ is a discrete topological space, establish that $\{\{x\} : x \in X\}$ is a basis.
4. If $\mathbb{R}$ is the set of real numbers and $(\mathbb{R}, \Im_1)$ and $(\mathbb{R}, \Im_2)$ are the usual and discrete topologies respectively on $\mathbb{R}$, is there any continuous function from $(\mathbb{R}, \Im_1)$ to $(\mathbb{R}, \Im_2)$. Justify your answer.
5. If $X = \{a, b, c\}$ and $\Im = \{\phi, \{a\}, \{c\}, \{a, b\}, \{a, c\}, X\}$, is $(X, \Im)$ a T_2 space. Justify your answer.

SECTION – B

Answer **any six** questions out of the nine questions. **Each** question carries **2** marks. **(6×2=12)**

1. Define a dense subset in a topological space and give an example.
2. Let $(X, \Im)$ be a topological space and Y be a non empty subset of X. If D is a dense subset of $(X, \Im)$, is it always true that $D \cap Y$ is dense in $(Y, \Im/Y)$? Justify.

3. Define a compact topological space and given an example for the same.

4. Is it true that every compact subset of a topological space is closed ? Justify.

5. Let $(X, \Im_1)$ and $(X, \Im_2)$ be two topologies on X. If $\Im_2$ is weaker than $\Im_1$, prove that the identity function $I : X \to X$ is continuous from $(X, \Im_1)$ to $(X, \Im_2)$.

6. Prove that second countability is a hereditary property.

7. Give an example of a T_1 space which is not T_2.

8. Define a completely regular topological space.

9. Are there any non empty disjoint open subsets C and D in $\mathbb{R}$ with usual topology such that $C \cup D = \mathbb{R}$? Justify.

SECTION – C

Answer **any eight** questions out of the twelve questions. **Each** question carries 4 marks. **(8×4=32)**

1. If $(X, \Im_1)$ and $(X, \Im_2)$ are topological spaces on a set X with bases β_1 and β_2 respectively, prove that $\Im_1$ is weaker than $\Im_2$ if and only if every member of β_1 is a union of members of β_2.

2. Let $(X, \Im)$ be a topological space and A, B are subsets of X. Prove the following :
   i) $\bar{A}$ is the smallest closed set containing A
   ii) A is closed in X if and only if $\bar{A} = A$
   iii) $\overline{A \cup B} = \bar{A} \cup \bar{B}$.

3. Let $(X, \Im_1)$ and $(Y, \Im_2)$ be two topological spaces and every function from $X \to Y$ is continuous from $(X, \Im_1)$ to $(X, \Im_2)$. Prove that either $\Im_1$ is discrete or $\Im_2$ is indiscrete.

4. Let (X, d) be metric space and $A \subset X$. Prove that $x \in \bar{A}$ if and only if there is a sequence $\{x_n\}$ in A such that $\{x_n\}$ converges to x.

5. Define a quotient map and prove that every open surjective map is a quotient map.

6. Let $(X, \Im_1)$ be a topological space and let $\Im_2$ be the discrete topology on $Y = \{a, b\}$. Prove that $(X, \Im_1)$ is disconnected if and only if there is a non constant continuous function from $(X, \Im_1)$ to $(Y, \Im_2)$.

7. Prove that a space is connected if and only if it cannot be written as the disjoint union of two non empty closed subsets.

8. Define a path connected space and prove that a path connected space is connected.

9. Prove that every infinite subset A of a compact space X has at least one accumulation point in X.

10. Prove that in a Hausdorff space $(X, \Im)$, a point x in X and a compact set F not containing x can be separated by disjoint open sets and establish that F is closed.

11. a) If S^1 is the unit circle, is there any continuous bijection from S^1 to $\mathbb{R}$? Justify your answer.
    b) What is the smallest topology $\Im$ on a set X such that $(X, \Im)$ is a T_1 space ?

12. Define a regular space. Prove that in a regular space $(X, \Im)$, for any $x \in X$ and any open set G containing x, there is an open set H such that $x \in H \subset \bar{H} \subset G$.

SECTION – D

Answer **any two** questions from the four questions. **Each** question carries **6** marks. **(2×6=12)**

1. a) Let $(X, \Im)$ be a topological space and A be a subset of X. Show that $\bar{A} = \{y \in X :$ every neighborhood of y meets A non vacuously$\}$
   b) If $(X, \Im)$ is a discrete topological space, is there any proper dense sub set of X ? Justify your answer.

2. a) Let $\mathcal{C}$ be a collection of connected subsets of a space X such that no two members of $\mathcal{C}$ are mutually separated. Prove that $\cup\{c : c \in \mathcal{C}\}$ is connected.
   b) If X_1 and X_2 are two connected topological spaces. Prove that $X_1 \times X_2$ is connected in the product topology.

3. a) Prove that every continuous function from a compact space into a T_2 space is closed.
   b) Prove that continuous bijection from a compact space into a Hausdorff space is a homeomorphism.

4. Let $(X, \Im)$ be an indiscrete space. Then prove the following :
   i) $(X, \Im)$ is a normal space
   ii) Every continuous function from X to [0, 1] is a constant
   iii) Do (i) and (ii) together contradict the Urysohn's lemma ? Justify your answer.

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