K21U 0217

Reg. No. : ....

Name : ....

VI Semester B.Sc. Hon's (Mathematics) Degree (Supple.)
Examination, April 2021
(2014 – 2015 Admissions)
BHM601 – MEASURE AND INTEGRATION

Time : 3 Hours Max. Marks : 80

Answer **all** the **10** questions. **(1×10=10)**

1. Define simple function.
2. Find lim sup x_n where sequence $(x_n) = (n)$.
3. Define σ algebra.
4. Give an example of measurable space.
5. Define Borel Set.
6. Define measure.
7. Give an example of σ finite measure.
8. Write the integral value of characteristic function defined on the interval [1, 5].
9. Define Pseudo norm.
10. State Holders Inequality.

Answer **any 10** short answer questions out of 14. **(3×10=30)**

11. Show that a characteristic function defined on a set E is measurable if and only if E is X-measurable set.
12. Give an example of a σ-algebra on the set X = {1, 2, 3, 4, 5}.
13. Show that if f and g are real valued measurable function then f + g and f g are measurable.

P.T.O.

14. Let (f_n) be the sequence of extended real valued measurable function and it converges to f. Then show that f is also real valued extended measurable function.

15. Let μ be a measure defined on σ algebra X. If E and F belongs to X with $E \subset F$ and $\mu(E) < \infty$ then show that $\mu(F/E) = \mu(F) - \mu(E)$.

16. Show that a measurable function f belongs to L if and only if |f| belongs to L.

17. Define Counting Measure.

18. State and prove Minkowski's Inequality.

19. If f belongs to M^+, show that $f(x) = 0$ if and only if $\int f d\mu = 0$.

20. State and prove Riesz-Fischer theorem.

21. Show that if the sequence (f_n) converges to f in L_p then it is a cauchy sequence.

22. If $f \in L_p$ and $g \in L_q$ where $p > 1$ and $(1/p) + (1/q) = 1$. Then show that $f g \in L_1$.

23. Let (g_n) be a sequence in M^+, then $\int\left(\sum_{n=1}^{\infty}\right)g_n d\mu = \sum_{n=1}^{\infty}\left(\int g_n d\mu\right)$.

24. Define the linear space L_∞ also show that if $f \in L_\infty$ then $|f(x)| \leq \|f\|$.

Answer **any 6** short essay questions out of 9. **(6×5=30)**

25. If f is a non negative function in $M(X, \mathbf{X})$ then their exist a sequence (ϕ_n) in $M(X, \mathbf{X})$ such that
    a) $0 \leq \phi_n(x) \leq \phi_{n+1}(x)$ for $x \in X, n \in \mathbb{N}$
    b) $f(x) = \lim \phi_n(x)$ for each $x \in X$
    c) Each ϕ_n has only a finite number of real values.

26. Let μ be measure defined on σ-algebra X. Then show that if (E_n) is a increasing sequence in X then $\mu\left(\bigcup_{n=1}^{\infty} E_n\right) = \lim \mu(E_n)$.

27. Let μ be a measure on X and A is a fixed set in X then show that the function λ defined on X defined by $\lambda(E) = \mu(E \cap A)$ is a measure.

28. If ϕ and ψ are two measurable functions in $M^+(X, \mathbf{X})$ then show that $\int(\phi + \psi)d\mu = \int \phi d\mu + \int \psi d\mu$.

29. State and prove monotone convergence theorem.

30. State and prove Fatous Lemma.

31. If f belongs to M^+ and if λ is defined on X by $\lambda(E) = \int_E f d\mu$, then show that λ is a measure.

32. Suppose that for some $t_0 \in [a, b]$, the function $x \to f(x, t_0)$ is integrable on X, that $\partial f/\partial t$ exists on $X \times [a, b]$ and that there exist an integrable function g on X such that $\left|\frac{\partial f}{\partial t}(x, t)\right| \leq g(x)$ and if $F(t) = \int f(x, t) d\mu(X)$. Show that $\frac{dF}{dt}(t) = \frac{d}{dt}\int f(x, t) d\mu(x)$.

33. Show that the linear space L_∞ is a completed normed space.

Answer **any one** question out of 2. **(1×10=10)**

34. State and prove Lebesgue Dominated Convergence theorem.

35. If $1 \leq p < \infty$ then show that the space L_p is a complete linear space under the norm $\|f\|_p = \left\{\int |f|^p d\mu\right\}^{\frac{1}{p}}$.

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