SECTION – D

(Answer **any 2** questions out of 6 questions. **Each** question carries **6** marks.) **(2×6=12)**

37. a) Use the convolution theorem to show that $Z^{-1}\left\{\frac{z(z+1)}{(z-1)^3}\right\} = n^2$.

    b) Solve the initial value problem for the difference equation $f(n+1) - f(n) = 1$, $f(0) = 0$.

38. Solve $y'' + y = 2t$, $y(\pi/4) = \pi/2$, $y'(\pi/4) = 2 - \sqrt{2}$.

39. Solve the second order difference equation $u_{n+2} - 2xu_{n+1} + u_n = 0$, $|x| \le 1$, $u(0) = u_0$, $u(1) = u_1$, where u_0 and u_1 are constants.

40. Suppose f(x) and g(x) are piece-wise continuous, bounded and absolutely integrable on the x-axis. Then show that $\mathcal{F}(f * g) = \sqrt{2\pi}\, \mathcal{F}(f) . \mathcal{F}(g)$.

41. State and prove translation property of Z-transforms.

42. State and prove second shifting theorem for Laplace transforms.

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, April 2021
(2016 Admission Onwards)

BHM601 : MATHEMATICAL TRANSFORMS

Time : 3 Hours Max. Marks : 60

SECTION – A

(Answer **any 4** questions out of 8 questions. **Each** question carries **1** mark.) **(4×1=4)**

1. Let f(t) = 1 when $t \ge 0$. Find $\mathcal{L}(f)$.
2. State the convolution theorem for Laplace transforms.
3. Find the first order Hankel transform of $f(r) = r\exp(-ar^2)$.
4. Define Fourier cosine transform of an even function f(x).
5. Define Z-transform of a sequence {f(n)} as the function F(z) of a complex variable z.
6. Let f(t) and g(t) be two functions such that their Laplace transforms exist. Let a and b be any constants. Then $\mathcal{L}(af(t) + bf(t)) = \ldots.$
7. Find $Z^{-1}\{e^{1/z}\}$.
8. Find z{1}.

SECTION – B

(Answer **any 6** questions out of 12 questions. **Each** question carries **2** marks.) **(6×2=12)**

9. Find the Laplace transform of sinh at.
10. Find the Fourier sine transform of the function.

$$f(x) = \begin{cases} k & , \text{if } 0 \le x \le a \\ 0 & , \text{if } x > a \end{cases}$$

11. Find the inverse Z-transform of $F(z) = z/(z^2 - 6z + 8)$.

12. State shifting property of the Mellin transform.

13. Let $f(n) = n$. Find $Z\{n\}$.

14. Find the inverse transform of $1/(s(s^2 + w^2))$, using Laplace transform of integral.

15. Verify the initial value theorem for the function

$$F(z) = \frac{z}{(z-a)(z-b)}$$

16. Find the Mellin cosine transform of the function $f(x) = e^{-nx}$, where $n > 0$.

17. Show that $\tilde{f}(k) = \mathcal{H}\left\{\frac{e^{-ar}}{r}\right\} = (1/k)[1 - a(k^2 + a^2)^{-1/2}]$.

18. State and prove scaling property of Mellin transform.

19. Let $f(n) = \frac{1}{n!}$. Find $z\left\{\frac{1}{n!}\right\}$.

20. Let $f(n) = a^n$, $n \geq 0$. Find $Z\{a^n\}$.

SECTION – C

(Answer **any 8** questions out of 16 questions. **Each** question carries **4** marks.) **(8×4=32)**

21. Solve the initial value problem $y' + (1/2)y = 17\sin(2t)$, $y(0) = -1$, using the Laplace transform.

22. State and prove linearity property of Fourier transform.

23. If $f(x) = (e^x - 1)^{-1}$. Find $\mu\{1/(e^x - 1)\}$.

24. Show that, $\mu\left\{\frac{1}{(1+x)^n}\right\} = \frac{\Gamma(p)\Gamma(n-p)}{\Gamma(n)}$.

25. Find n^{th} order $(n > -1)$ Hankel transform of $f(r) = r^n e^{-ar^2}$.

26. Solve the initial value problem $u(n + 2) - u(n + 1) + u(n) = 0$, $u(0) = 1$, $u(1) = 2$.

27. Let $f(x)$ be continuous on the x-axis and $f(x) \to 0$ as $|x| \to \infty$. Furthermore, let $f'(x)$ be absolutely integrable on the x-axis. Then show that $\mathcal{F}(f'(x)) = iw\mathcal{F}\{f(x)\}$.

28. Use Z-transforms to show that $\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$.

29. If $f(n) = \cosh nx$, then show that $Z\{\cosh nx\} = \frac{z(z - \cosh x)}{z^2 - 2z\cosh x + 1}$.

30. Represent $f(x)$ as a Fourier cosine integral, $f(x) = \begin{cases} 1 & , \text{if } 0 < x < a \\ 0 & , \text{if } x > 1 \end{cases}$.

31. State and prove initial value problem for Z-transforms.

32. Find the Fourier transform of $f(x) = \begin{cases} e^{-ax} & , \text{if } x > 0 \\ 0 & , \text{if } x < 0 \end{cases}$. Here $a > 0$.

33. Solve the equation $f(n + 1) + 2f(n) = n$, $f(0) = 1$ using Z-transform.

34. Find the sum of the series $\sum_{n=0}^{\infty} a^n \sin nx$.

35. Find the inverse transform of $F(s) = \frac{e^{-s}}{s^2 + \pi^2} + \frac{e^{-2s}}{s^2 + \pi^2} + \frac{e^{-3s}}{(s+2)^2}$.

36. If $\mathcal{H}_n\{f(r)\} = \tilde{f}_n(k)$, then show that $\mathcal{H}_n\{f(ar)\} = \frac{1}{a^2}\tilde{f}_n\left(\frac{k}{a}\right)$, $a > 0$.