K20U 0213

Reg. No. : ....

Name : ....

VI Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, April 2020
(2016 Admission Onwards)
BHM 601 : MATHEMATICAL TRANSFORMS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark. **(4×1=4)**

1. Let f(t) = 1 when $t \geq 0$. Find $\mathscr{L}(f)$.
2. Let f(t) and g(t) be two functions such that their Laplace transforms exist. Let a and b be any constants. Then $\mathscr{L}(af(t) + b\, f(t))$ = ...
3. Define Fourier cosine transform of an even function f(x).
4. Obtain the zero order Hankel transform of $r^{-1}.\exp(-ar)$.
5. Find z{1}.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks. **(6×2=12)**

6. State and prove linearity property of Laplace transform.
7. Find the Laplace transform of sinh at.
8. Find the Fourier cosine transform of the function

$$f(x) = \begin{cases} k, & \text{if } 0 \leq x \leq a \\ 0, & \text{if } x > a \end{cases}$$

9. Find the first order of Hankel transform of $f(r) = \frac{1}{r}e^{-ar}$.
10. Find the Mellin transform of the function f(x) = 1/(1 + x).

11. Let $f(n) = \frac{1}{n!}$. Find $z\left\{\frac{1}{n!}\right\}$.

12. Verify the initial value theorem for the function

$$F(z) = \frac{z}{(z-a)(z-b)}.$$

13. Find the inverse transform of $F(z) = z(z-a)^{-1}$.

14. State shifting property of the Mellin transform.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Solve the initial value problem $y'' - y = t,\ y(0) = 1, y'(0) = 1$, using the Laplace transform.

16. State and prove convolution theorem for Laplace transforms.

17. Solve the Voltera integral equation

$$y(t) = \int_0^t (1+\tau)\, y(t-\tau)\, d\tau = 1 - \sinh t$$

18. Find the Fourier integral representation of the function

$$f(x) = \begin{cases} 1, & \text{if } |x| < 1 \\ 0, & \text{if } |x| > 1 \end{cases}$$

19. Find the Fourier cosine transform $F_c(e^{-ax})$, where $a > 0$.

20. Find the Fourier transform of xe^{-x^2}.

21. Find the n^{th} $(n > -1)$ order Hankel transform of $f(r) = r^n \exp(-ar^2)$.

22. Show that if $\mu[f(x)] = \bar{f}(p)$ and $\mu\{g(x)\} = \bar{g}(p)$, then,

$$\mu[f(x)g(x)] = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \bar{f}(s)\bar{g}(p-s)\, ds$$

23. If $f(x) = \cosh nx$, then show that $Z\{\cosh nx\} = \frac{z(z-\cosh x)}{z^2 - 2z\cos hx + 1}$.

24. State and prove the initial value theorem for z-transforms.

25. Solve the equation $f(n+1) + 2f(n) = n,\ f(0) = 1$ using z-transform.

26. Use the z-transform to show that $\sum_{n=0}^{\infty} \frac{x^n}{n!} = e^x$.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. State and prove second shifting theorem for Laplace transforms.

28. Suppose f(x) and g(x) are piece-wise continuous, bounded and absolutely integrable on the x-axis. Then show that

$$F(f * g) = \sqrt{2\pi}\, F(f).F(g)$$

29. If $H_n\{f(r)\} = \tilde{f}_n(k)$, then show that

$$H_n\left\{\left(\nabla^2 - \frac{n^2}{r^2}\right) f(r)\right\} = H_n\left\{\frac{1}{r}\frac{d}{dr}\left(r\frac{df}{dr}\right) - \frac{n^2}{r^2} f(r)\right\} = -K^2 \tilde{f}_n(k),$$

provided both $r f'(r)$ and $r f(r)$ vanish as $r \to 0$ and $r \to \infty$.

30. Use z-transforms to show that

$$\sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1} = \log(1+x).$$

____