K19U 0761

Reg. No. : ....

Name : ....

VI Semester B.Sc. Hon's (Mathematics) Degree (Reg.) Examination, April 2019
(2016 Admission)
BHM 601 : MATHEMATICAL TRANSFORMS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries 1 mark. **(4×1=4)**

1. Find $\mathscr{L}\{u(t-a)\}$.
2. Find the Laplace transform of $e^{at}\sin\omega t$.
3. Find the zero-order Hankel transform of $\frac{\delta(r)}{r}$.
4. Show that $\mathscr{M}\{f(ax)\} = a^{-p}\tilde{f}$ $a > 0$.
5. Evaluate $Z\left\{\frac{1}{n!}\right\}$.

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries 2 marks. **(6×2=12)**

6. Find the inverse Laplace transform of $\frac{4s-2}{s^2-6s+18}$.
7. Find the Laplace transform of $f(t) = t\cosh 2t$.
8. By using Laplace transform, solve $y'' - y' - 6y = 0$, $y(0) = 11$ and $y'(0) = 28$.
9. Find the Fourier sine transform of $f(x) = e^{-ax}$, where $a > 0$.
10. Show that if $\mathscr{H}_n\{f(r)\} = \tilde{f}(k)$ then $\mathscr{H}_n\{f(ar)\} = \frac{1}{a^2}\tilde{f}\left(\frac{k}{a}\right)$, where $a > 0$.

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11. Show that if $\mathcal{M}\{f(x)\} = \bar{f}(p)$ then $\mathcal{M}\left\{\frac{1}{x} f\left(\frac{1}{x}\right)\right\} = \bar{f}(1-p)$.

12. Find the Mellin transform of sin kx.

13. Find the Z-transform of $f(n) = na^n$.

14. Find the inverse Z-transform of $F(z) = \frac{z}{z^2 - 6z + 8}$.

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks. **(8×4=32)**

15. Find the inverse Laplace transform of $\frac{\omega}{s^2(s^2+\omega^2)}$.

16. Solve the system of equations $y'_1 = -y_1 + 4y_2$, $y'_2 = 3y_1 - 2y_2$, $y_1(0) = 3$ and $y_2(0) = 4$ using Laplace transforms.

17. Solve the integral equation $y(t) - \int_0^t y(\tau)d\tau = 1$.

18. Represent $f(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$ as a Fourier integral.

19. Find the Fourier sine transform of $f(x) = e^{-\pi x}$.

20. Find the n^{th} $(n > -1)$ order Hankel transform of $f(r) = r^n e^{-ar^2}$.

21. State and prove Parseval's relation in Hankel transforms.

22. Find $\mathcal{M}\{f(x)\}$, where $\frac{1}{(1+x)^n}$.

23. Show that if $\mathcal{M}\{f(x)\} = \bar{f}(p)$ then $\mathcal{M}\{(\log x)^n f(x)\} = \frac{d^n}{dp^n}\bar{f}(p)$.

24. Evaluate Z{cos nx} and Z{sin nx}.

25. Using convolution evaluate the inverse Z-transform of $\frac{z^2}{(z-a)(z-b)}$.

26. Solve the difference equation $f(n+1) - af(n) = a^n$, $f(0) = x_0$.

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks. **(2×6=12)**

27. Solve $y'' + 3y' + 2y = u(t-1) + u(t-2)$, $y(0) = y'(0) = 0$.

28. Using Fourier integral, show that

$$\int_0^\infty \frac{\cos x\omega + \omega \sin x\omega}{1+\omega^2} d\omega = \begin{cases} 0 & \text{if } x < 0 \\ \frac{\pi}{2} & \text{if } x = 0 \\ \pi e^{-x} & \text{if } x > 0 \end{cases}$$

29. Let $\mathcal{M}\{f(x)\} = \bar{f}(p)$ and $\mathcal{M}\{g(x)\} = \bar{g}(p)$. Show that

$\mathcal{M}[f(x) * g(x)] = \bar{f}(p)\,\bar{g}(p)$ and

$\mathcal{M}[f(x) \circ g(x)] = \bar{f}(p)\,\bar{g}(1-p)$

30. a) Solve the difference equation

$f(n+2) + 4f(n+1) + 3f(n) = 0$, $f(0) = 1$, $f(1) = 1$.

b) Show that if $Z\{f(n)\} = F(z)$ then $Z\{na^n f(n)\} = -z\frac{d}{dz}\left\{F\left(\frac{z}{a}\right)\right\}$.
