K21U 0218

Reg. No. : ....

Name : ....

VI Semester B.Sc. Hon's (Mathematics) Degree (Supple.) Examination, April 2021
(2014-2015 Admissions)
BHM 602 : INTEGRAL EQUATIONS AND TRANSFORMS

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark : **(10×1=10)**

1. Define Volterra Integral equation of the third kind.
2. Define Fredholm equation of second kind.
3. Define convolution.
4. Define symmetric kernel.
5. Define Abel's equation.
6. Find L(3sin2t + 4cos3t).
7. Write the linearity property of laplace transform.
8. Define Dirac Delta function.
9. Define fourier sine transform.
10. Define inverse fourier cosine transform.

SECTION – B

Answer **any 10** questions. **Each** question carries **3** marks : **(10×3=30)**

11. Show that the function $\phi(x) = 1$ is a solution of the Fredholm integral equation $\phi(x) + \int_0^1 x e^{(x\xi - 1)} \phi(\xi)\, d\xi = e^x - x$.

P.T.O.

12. Explain briefly the procedure for solving a Fredholm integral equation of second kind with separable integral.

13. Solve the homogeneous Fredholm integral equation $\phi(x)=\lambda\int_0^1 e^x e^\xi \phi(\xi)\,d\xi$.

14. Find the first two iterated kernels of the kernel $K(x, t) = (x - t)^2$, $a = -1$, $b = 1$.

15. Write short note on Abel's equation.

16. Find the eigen values and eigen functions of $y(x)=\lambda\int_0^{2\pi}\sin x\cos t\,y(t)\,dt$.

17. Find $L[e^{5t}\cos 3ht]$.

18. Find $L^{-1}\left(\frac{1}{s^2\left(s^2+a^2\right)}\right)$.

19. Find the inverse Laplace Transform of the function $\frac{1}{s^2-4s+5}$.

20. State and prove first shifting theorem.

21. Find $L[t\cos 3t]$.

22. State and prove linearity property of Fourier Transform.

23. Find the Fourier transform of $f(x) = 1$ if $|x| < 1$ and $f(x) = 0$ otherwise.

24. Find the Fourier cosine transformation of the function $f(x)=\begin{cases} k & \text{if } 0<x<a \\ 0 & \text{otherwise}\end{cases}$.

SECTION – C

Answer **any 6** questions. **Each** question carries **5** marks : **(6×5=30)**

25. Form the Fredholm integral equation corresponding to the boundary value problem $y'' = f(x)$, $y(0) = 0$, $y(1) = 0$.

26. Find the Green's function of the boundary value problem $\frac{d^4y}{dx^4}+\lambda y=-f(x)$ with $y(0) = y'(0) = 0$, $y(1) = y'(1) = 0$.

27. If the Kernel K (x, t) is real and symmetric then show that the eigen function corresponding to distinct eigen values of the homogenous Fredholm integral equation $y(x)=\lambda\int_a^b k(x,t)\,y(t)\,dt$ are orthogonal.

28. Find eigen value and eigen function of $y(x)=\lambda\int_0^1\left(\sqrt{x}t-\sqrt{t}x\right)y(t)\,dt$.

29. Solve the initial value problem : $\frac{d^2y}{dt^2}+2\frac{dy}{dt}-3y=\sin t$ given that $y(0) = y'(0) = 0$.

30. Solve $y'' - 3y' + 2y = 4t$, $y(0) = 1$, $y'(0) = -1$.

31. Find the inverse Laplace transform of $ln\,\frac{s+a}{s-a}$.

32. Find the Fourier Transform of $f(x) = e^{-ax}$, if $x > 0$, $f(x) = 0$, if $x < 0$, where $a > 0$.

33. State and prove convolution theorem of fourier transform.

SECTION – D

Answer **any one** question. It carries **10** marks. **(1×10=10)**

34. a) Write down the four properties that have to be satisfied by Green's function of a second order differential equation with homogenous boundary conditions.

b) Define singular integral equations and give examples for each case.

35. a) Evaluate $\int_0^\infty \frac{e^{-t}\sin t}{t}\,dt$.

b) Find the Fourier integral representation of $f(x)=\begin{cases}\sin x & \text{if } 0<x<\pi \\ 0 & \text{if } x>\pi\end{cases}$.