K16U 1277

Reg. No. : ....

Name : ....

VI Semester B.Sc. Hon's (Mathematics) Degree (Regular)
Examination, May 2016
BHM 602 : INTEGRAL EQUATION AND TRANSFORMS

Time : 3 Hours
Max. Marks : 80

Answer **all** the **ten** questions.
·(10×1=10)

1. Define Fredholm Integral Equation.

2. Define Orthogonality of two functions.

3. Define singular integral equations.

4. Define Wronskian.

5. Define characteristic function of the Integral Equation.

6. Final $Z \, (e^{at} \cos \omega t)$.

7. Using convolution theorem find the inverse of $\left(\dfrac{1}{S^3}\right)$.

8. Find the Laplace Transform of $(2t + 6)$.

9. Define Fourier Integral.

10. Find the Fourier Transform of $f(x) = \begin{cases} K & \text{if } 0 < x < a \\ 0 & \text{otherwise} \end{cases}$

Answer **any 10** short answer questions out of 14.
(10×3 = 30)

11. Show that the function $\phi(x) = 1 - x$ is a solution of the integral equation

$$\int_0^x e^{x-t} \phi(t)\, dt = x$$.

P.T.O.
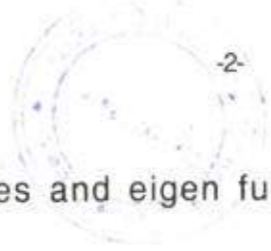
12. Find the eigen values and eigen function for the integral equation

$$y(x) = \lambda \int_0^{\pi/4} \sin^2 x \, y(t) \, dt$$

13. Find the resolvant Kernal for $K(x, t) - xe^t$; $a = -1, b = 1$.

14. Solve the Integral Equation $\phi(x) = e^x - \dfrac{e}{2} + \dfrac{1}{2} + \dfrac{1}{2} \int_0^1 \phi(t) dt$.

15. Find the iterated Kernal of the Kernal $K(x,t) = e^x \cos t$; $a = 0, b = \pi$.

16. Solve the Fredholm Integral Equation $y(x) = \lambda \int_0^1 (1 - 3xt) y(t) dt + x$.

17. Find the Laplace Transform of $L(t \sin at)$.

18. Find the Inverse Laplace Transform of $\left(\dfrac{3S}{S^2 + 2S - 8} \right)$.

19. State and prove Linearity of the Laplace Transform.

20. Derive the Laplace Transform of $\cos \omega t$.

21. Let $L(f) = \dfrac{1}{S(s^2 + \omega^2)}$. Find $f(b)$.

22. Find the Fourier Integral representation of the function $f(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{if } |x| > 1 \end{cases}$

23. Prove the Linearity property of sine transforms.

24. Find the Fourier Transform of xe^{-x^2}.

Answer **any 6** short essay questions out of 9.   (6×5 = 30)

25. Form Integral Equation corresponding to $y'' + xy = 1$; $y(0) = 0, y(1) = 1$.

26. Solve $(x) = \cos x + \lambda \int_0^\pi \sin x \, \phi(t) dt$.

27. Find the eigen values and eigen functions of $y(x) = \lambda \int_0^{2\pi} \sin(x + t) y(t) dt$.

28. Find the Green's function for the boundary value problem $\dfrac{d^2 y}{dx^2} - y(x) = 0$ with

    $y(0) = y(1) = 0$.

29. Solve $y'' - y = t$; $y(0) = 1, y'(0) = 1$.

30. State and prove convolution theorem.

31. Solve $y'' + y' - 6y = 1$, $y(0) = 0$, $y'(0) = 1$.

32. Find the Fourier cosine and sine integrals of $f(x) = e^{-kx}$ $(x > 0, k > 0)$.

33. State and prove Fourier transform of the derivative of $f(x)$.

Answer **any one** essay question out of 2.   (1×10 = 10)

34. Solve the Integral equation $\phi(x) = \cos 3x + \lambda \int_0^\pi \cos(x + t)\phi(t) dt$.

35. 1) Find the Fourier cosine integral of $f(x) = \begin{cases} 1 & \text{if } 0 < x < 1 \\ 0 & \text{if } x > 1 \end{cases}$

    ii) Find $F_s(e^{-ax})$; $a > 0$ by integration.

    _____