



K19U 0776

Reg. No. :

Name :

**VI Semester B.Sc. Hon's (Mathematics) Degree (Supplementary)
Examination, April 2019
(2013 – 15 Admissions)
BHM 602 : INTEGRAL EQUATIONS AND TRANSFORMS**

Time : 3 Hours

Max. Marks : 80

Answer **all** the **ten** questions.

(10×1=10)

1. Define Volterra integral equation of second kind.
2. Define symmetric Kernel of the integral equation with an example.
3. Define iterated kernels of Fredholm integral equation of the second kind.
4. Define Abel's integral equation.
5. Define singular integral equation with an example.
6. Find $L\{\cos at\}$.
7. Find the Laplace transform of $\sin^3 2t$.
8. Using the convolution theorem, find $L^{-1}\left\{\frac{s}{(s^2 + a^2)^2}\right\}$.
9. By using second shifting property find the Laplace transform of $(t - 1)^2 u(t - 1)$.
10. Find the Fourier sine transform of $\frac{1}{x}$.

Answer **any 10** short answer questions out of 14 :

(10×3=30)

11. Convert the given integral equation $y(x) = \int_0^x (x + t) y(t) dt + 1$ into a differential equation with initial conditions.
12. Solve the integral equation $x = \int_0^x e^{x-t} \phi(t) dt$.

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13. Show that the eigen values of a symmetric Kernel are real.
14. Find the first two iterated kernels of the kernel $K(x, t) = (x - t)^2$; $a = -1$, $b = 1$.
15. Find the resolvent kernel of the Volterra integral equation with kernel $k(x, t) = 1$.

16. Solve the integral equation $x = \int_0^x e^{(x-t)} y(t) dt$.

17. Find the Laplace transform of $\frac{1 - \cos t}{t^2}$.

18. Find the Laplace transform of $t^2 e^{-2t} \cos t$.

19. State and prove convolution theorem for Laplace transforms.

20. Find the inverse Laplace transform of $\frac{5s + 3}{(s - 1)(s^2 + 2s + 5)}$.

21. Let $H(s) = \frac{s^2}{(s^2 + b^2)(s^2 + a^2)}$. Then find $L(t)$.

22. Find the Fourier sine transform of $e^{-|x|}$.

23. Find the Fourier cosine transform of $f(x) = e^{-2x} + 4e^{-3x}$.

24. Using Parseval's identity show that $\int_0^{\infty} \frac{x^2 dx}{(x^2 + 1)^2} = \frac{\pi}{4}$.

Answer **any 6** short answer questions out of 9.

(6×5=30)

25. Convert the differential equation $y''(x) - 3y'(x) + 2y(x) = 5 \sin x$, $y(0) = 1$, $y'(0) = -2$ integral equation.

26. Find the Green's function for the boundary value problem $y'' + \mu^2 x = 0$, $y(0) = 0 = y(1)$.

27. Solve the integral equation $u(x) = \cos x + \lambda \int_0^x \sin(x - t) u(t) dt$.

28. If the Kernel $K(x, t)$ is real and symmetric then the eigen functions corresponding to distinct eigen values of the homogeneous Fredholm integral equation

$$y(x) = \lambda \int_a^b k(x, t) y(t) dt$$

are orthogonal.



29. Solve $y'' + 2y' + 5y = e^{-x} \sin x$, where $y(0) = 0$ and $y'(0) = 1$ using the Laplace transforms.

30. Solve $y''' + 2y'' - y' - 2y = 0$ given $y(0) = y'(0) = 0$ and $y''(0) = 6$ using the Laplace transforms.

31. Solve for $f(x)$ from the integral equation $\int_0^{\infty} f(x) \cos sx dx = e^{-s}$.

32. State and prove shifting property and modulation theorem of the complex Fourier Transform.

33. Find the Fourier cosine transform of the function

$$f(x) = \begin{cases} x, & \text{for } 0 < x < 1/2 \\ 1 - x, & \text{for } 1/2 < x < 1 \\ 0 & \text{for } x > 1 \end{cases}$$

Answer **any one** essay question out of 2.

(1×10=10)

34. Solve $u(x) = f(x) + \lambda \int_0^1 (1 - 3xt) u(t) dt$ with separable kernel and discuss all possible cases.

35. i) Find the Fourier sine and cosine transform of x^{n-1} and $\frac{1}{\sqrt{|x|}}$.

ii) Find the Fourier transform of $f(x) = \begin{cases} 1 - x^2, & \text{when } |x| \leq 1 \\ 0, & \text{when } |x| > 1 \end{cases}$; hence evaluate $\int_0^{\infty} \frac{x \cos x - \sin x}{x^3} \cos \frac{x}{2} dx$.