K16U 1280

Reg. No. : ....

Name : ....

VI Semester B.Sc. Hon's (Mathematics) Degree (Regular) Examination, May 2016

BHM 605 (A) : DISCRETE FOURIER ANALYSIS

Time : 3 Hours

Max. Marks : 80

Answer **all** the **ten** questions. **(10×1=10)**

1. Write Parseval's relation in $l^2(\mathbb{Z}_N)$.
2. Define translation invariant linear transformation in $l^2(\mathbb{Z}_N)$.
3. What do you mean by convolution operator in $l^2(\mathbb{Z}_N)$?
4. What do you mean by conjugate reflection of $\omega \in l^2(\mathbb{Z}_N)$?
5. If z = (2, 5, – 1, i), find U(z).
6. Define orthogonal direct sum of two subspaces of an inner product space.
7. What do you mean by square summable sequences ?
8. What do you mean by a trigonometric system ?
9. What is the first stage wavelet system for $l^2(\mathbb{Z})$?
10. Define homogeneous wavelet system for $l^2(\mathbb{Z})$.

Answer **any 10** short answer questions out of 14. **(10×3=30)**

11. What do you mean Fourier basis for $l^2(\mathbb{Z}_n)$?
12. Let $\omega = (2, 4 + 4i, -6, 4 - 4i) \in l^2(\mathbb{Z}_4)$. Find $\check{\omega}$.
13. Write an orthonormal basis for $l^2(\mathbb{Z}_4)$.

P.T.O.

14. Prove that $(\bar{z})^\wedge(m)=\overline{\hat{z}(-m)}$ for $z \in l^2(\mathbb{Z}_N)$.

15. What do you mean by a circulant matrix ? Give an example.

16. Define $T : l^2(\mathbb{Z}_N) \to l^2(\mathbb{Z}_N)$ by $T(z)(n) = z(n+1) - 2z(n) + z(n-1)$. Find the eigen values of T.

17. Explain downsampling operator from $l^2(\mathbb{Z}_N)$ to $l^2(\mathbb{Z}_M)$.

18. Explain first stage Shannon wavelet basis in $l^2(\mathbb{Z}_N)$.

19. Suppose $N = 2M$, $z \in l^2(\mathbb{Z}_N)$ and $\omega \in l^2(\mathbb{Z}_{N/2})$. Then prove that $D(z) * \omega = D(z * U(\omega))$.

20. Explain convergence and absolute convergence of sequence of complex numbers.

21. What do you mean by Cauchy sequence in $l^2(\mathbb{Z})$? Give an example for a sequence in $l^2(\mathbb{Z})$ which is not Cauchy.

22. Define a norm and inner product on $L^2(-[\pi, \pi))$.

23. State Plancherel's formula and Parseval's formula in $L^2([-\pi, \pi))$.

24. What do you mean by translation operator and translation invariant linear transformation in $L^2([-\pi, \pi))$?

Answer **any 6** short answer questions out of 9. **(6×5=30)**

25. Define convolution in $l^2(\mathbb{Z}_N)$. Let $z = (1, 1, 0, 2)$ and $\omega = (i, 0, 1, i)$ be vectors in $l^2(\mathbb{Z}_4)$. Evaluate $z * \omega$.

26. Define $T : l^2(\mathbb{Z}_4) \to l^2(\mathbb{Z}_4)$ by $T(z)(n) = z(n) + 2z(n+1) + z(n+3)$. Find the eigen values and eigen vectors of T and diagonalize the matrix A representing T in the standard basis, if possible.

27. Let $\omega \in l^2(\mathbb{Z}_N)$. Then prove that $\{R_k\omega\}_{k=0}^{N-1}$ is an orthonormal basis for $l^2(\mathbb{Z}_N)$ if and only if $|\hat{\omega}(n)| = 1$ for all $n \in \mathbb{Z}_N$.

28. State and prove folding lemma.

29. Suppose $f : [-\pi, \pi] \to \mathbb{C}$ is continuous and bounded. If $\langle f, e^{in\theta} \rangle = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(\theta) e^{-in\theta}\, d\theta = 0$ for all $n \in \mathbb{Z}$, prove that $f(\theta) = 0$ for all $\theta \in [-\pi, \pi)$.

30. Suppose $T : L^2([-\pi, \pi)) \to L^2([-\pi, \pi))$ is a bounded, translation-invariant linear transformation. Then prove that for each $m \in \mathbb{Z}$, then there exists $\lambda_m \in \mathbb{C}$ such that $T(e^{im\theta}) = \lambda_m e^{im\theta}$.

31. Suppose that $z \in l^2(\mathbb{Z})$ and $\omega \in l^1(\mathbb{Z})$. Prove that $z * \omega \in l^2(\mathbb{Z})$ and $\| z * \omega \| \leq \|\omega\|_1 \|z\|$.

32. Suppose $\omega, z \in l^1(\mathbb{Z})$. Prove that the set $\{R_{2k}\omega\}_{k \in \mathbb{Z}}$ is orthonormal if and only if $|\hat{\omega}(\theta)|^2 + |\hat{\omega}(\theta + \pi)|^2 = 2$ for all $\theta \in [0, \pi)$.

33. Suppose that $u \in l^1(\mathbb{Z})$ and $\{R_{2k}u\}_{k \in \mathbb{Z}}$ is orthonormal in $l^2(\mathbb{Z})$. Define a sequence $\upsilon \in l^1(\mathbb{Z})$ by $\upsilon(k) = (-1)^{k-1}\overline{u(1-k)}$.
Show that $B = \{R_{2k}\upsilon\}_{k \in \mathbb{Z}} \cup \{R_{2k}u\}_{k \in \mathbb{Z}}$ is a complete orthonormal system in $l^2(\mathbb{Z})$.

Answer **any one** essay questions out of 2 . **(1×10=10)**

34. Suppose $z \in l^2(\mathbb{Z}_N)$ and $k \in \mathbb{Z}$. Prove that for any $m \in \mathbb{Z}$, $(R_k z)^\wedge(m) = e^{-2\pi imk/N}\hat{z}(m)$.

35. Briefly explain the construction of Daubechies's D_6 wavelets on $\mathbb{Z}_N$.
