K20U 0484

Reg. No. : ....

Name : ....

II Semester B.Sc. Degree (CBCSS (OBE) – Regular) Examination, April 2020
(2019 Admission)
CORE COURSE IN STATISTICS
2B02STA : Probability Theory and Mathematical Expectation

Time : 3 Hours Max. Marks : 48

PART – A

(Short Answer)

Answer **all 6** questions. **(6×1=6)**

1. Give axiomatic definition of probability.
2. If A and B are two mutually exclusive events such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{4}$. Find $P(A \cup B)$ and $P(A \cap B)$.
3. Define random variable with an example.
4. Define the probability mass function of a two-dimensional random variable.
5. Define moment generating function.
6. State the additive property of cumulants.

PART – B

(Short Essay)

Answer **any 7** questions. **(7×2=14)**

7. State and prove addition theorem on probability of two events.
8. If $P(A) = 0.5$, $P(B) = 0.3$ and $P(A \cap B) = 0.15$. Find $P(A \mid \bar{B})$.
9. Let X be a continuous random variable with probability density function $f(x) = \begin{cases} 6x(1-x), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$.

   Write down the distribution function of X and find $P\left(X \le \frac{1}{2} \middle| \frac{1}{4} \le X \le \frac{2}{3}\right)$.

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10. Define bivariate distribution function. Write down its properties.

11. If X and Y are two continuous random variables, then show that $E(X + Y) = E(X) + E(Y)$.

12. Define skewness of a random variable X. How will you evaluate and interpret the value of skewness of X ?

13. The moment generating function of a random variable X is $M(t) = \frac{pe^t}{1-qe^t}$. Find the mean and variance of X.

14. If $Y = aX + b$, then derive the relationship between the characteristic function between X and Y.

15. Define Probability Generating Function (PGF). How can we find mean and variance from PGF ?

PART – C

(Essay)

Answer **any 4** questions. **(4×4=16)**

16. A pair of dice are rolled once. A is the event that the first die has a 1 on it ; B is the event that the second die has a 6 on it; C is the event that the sum is 7. Check the independence of the events A, B and C.

17. The odds against manager X setting the wage dispute with the workers are 8 : 6 and odds in favour of manager Y setting the same dispute are 14 :16. Then (i) What is the probability that neither settles the dispute, if they both try, independently of each other ? (ii) What is the probability that the dispute will be settled ?

18. If the probability density function of continuous random variable X is given by

$$f(x) = \begin{cases} ax, & 0 \le x < 1 \\ a, & 1 \le x < 2 \\ 3a - ax, & 2 \le x < 3 \\ 0, & \text{otherwise} \end{cases}$$

Then find (i) the value of a (ii) the distribution function of X (iii) $P(X > 1.5)$.

19. Let X and Y be jointly distributed with PDF

$$f(x, y) = \begin{cases} \frac{1}{4}(1+xy), & |x| < 1, |y| < 1 \\ 0, & \text{otherwise} \end{cases}$$

Check the independence of X and Y.

20. If X and Y are two random variables, then show that $(E[XY])^2 \le E(X^2)E(Y^2)$.

21. Find the moment generating function of the random variable having PDF

$$f(x) = \begin{cases} x, & 0 \le x < 1 \\ 2 - x, & 1 \le x < 2 \\ 0, & \text{otherwise} \end{cases}$$

Hence find the mean and variance.

PART – D

(Long Essay)

Answer **any 2** questions. **(2×6=12)**

22. a) State and prove Baye's theorem.

b) The probabilities of X, Y and Z becoming managers are $\frac{4}{9}, \frac{2}{9}$ and $\frac{1}{3}$ respectively. The probabilities that the bonus scheme will be introduced if X, Y and Z becomes managers are $\frac{3}{10}, \frac{1}{2}$ and $\frac{4}{5}$ respectively. If the bonus scheme was introduced, what is the probability that the manager appointed was X ?

23. The joint PDF of (X, Y) is given by $f_{XY}(x, y) = \begin{cases} e^{-x-y}, & x > 0, y > 0 \\ 0, & \text{elsewhere} \end{cases}$. Let $U = \frac{X}{Y}$ and $V = X + Y$. Show that U and V are independent.

24. The distribution function of a continuous random variable X is given by $F(x) = 1 - (1 + x) e^{-x}, x \ge 0$. Find the moment measure of skewness and kurtosis.

25. Two random variables X and Y have the following joint probability density function

$$f(x, y) = \begin{cases} 2 - x - y, & 0 \le x \le 1, \ 0 \le y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

Find (i) marginal probability density functions of X and Y ; (ii) conditional density functions ; (iii) var(X) and Var(Y) and (iv) covariance between X and Y.

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