K18U 0320

Reg. No. : ....

Name : ....

VI Semester B.Sc. Hon's (Mathematics) Degree (Regular/Supple./Improv.) Examination, May 2018
BHM 604 : COMPLEX ANALYSIS – II
(2013 – 15 Admissions)

Time : 3 Hours Max. Marks : 80

Answer **all** the **ten** questions. (10×1=10)

1. What is the order of the pole z = 0 of $f(z) = \frac{1-e^{2z}}{z^4}$?
2. Define the term 'residue' of a complex function at a singular point.
3. State Cauchy's Residue theorem.
4. State Jordan's inequality.
5. What is the value of the winding number if Γ does not enclose the origin ?
6. State argument principle.
7. What do you mean by a linear transformation ?
8. Find the fixed points of the transformation $w = \frac{6z-9}{z}$.
9. What is the condition for the critical point of a transformation ?
10. State true or false : If f(z) = u(x, y) + iv(x, y) is an analytic function, then u(x, y) is the harmonic conjugate of v(x, y).

Answer **any 10** short answer questions out of 14. (10×3=30)

11. Find the residue of $f(z) = \frac{z^2+2}{z-1}$ at its pole.
12. What are the different types of singular points ? Give examples in each case.

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13. Prove that $\int_{|z|=1} e^{\frac{1}{z^2}} dz = 0$.

14. If z_0 is a pole of a function f, then prove that $\lim_{z \to z_0} f(z) = \infty$.

15. What are the singular points of $f(z) = e^{\frac{1}{z}}$? What is the type of these singular points ? Also find the residue of these singular points.

16. If C denote the unit circle $|z| = 1$, described in the positive sense, determine the winding number of $f(z) = \frac{z^3 + 2}{z}$.

17. Determine the number of zeros, counting multiplicities, of the polynomial $2z^4 - 2z^3 + 2z^2 - 2z + 9$ inside the circle $|z| = 1$.

18. What is the image of the infinite strip $0 < x < 1$ under the transformation $w = iz$?

19. Describe geometrically the transformation $w = \frac{i}{z}$.

20. Show that the transformation $w = iz + i$ maps the half plane $x > 0$ onto the half plane $v > 1$.

21. Prove that every linear fractional transformation can be expressed as a composition of linear transformations and inversion transformation.

22. What do you mean by a conformal mapping ? Give an example.

23. Find the angle of rotation at the point $z = 1$ for the transformation $w = \frac{1}{z}$.

24. Find the local inverse of the transformation $w = z^2$ at the point $z = 2$.

Answer **any 6** short answer questions out of 9. **(6×5=30)**

25. Prove that an isolated singular point z_0 of a function f is a pole of order m if and only if f(z) can be written in the form $f(z) = \frac{\phi(z)}{(z - z_0)^m}$, where $\phi(z)$ is analytic and nonzero at z_0.

26. Evaluate $\int_C \frac{3z^3 + 2}{(z-1)(z^2+9)} dz$, where C is the positively oriented circle $|z - 2| = 2$.

27. Suppose that a function f is analytic and bounded in some deleted neighborhood $0 \le |z - z_0| < \epsilon$ of a point z_0. If f is not analytic at z_0, then prove that it has a removable singularity there.

28. Using Residue theorem evaluate the integral $\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta}$.

29. Suppose that two functions f(z) and g(z) are analytic inside and on a simple closed contour C with $|f(z)| > |g(z)|$ at each point on C. Prove that f(z) and f(z) + g(z) have the same number of zeros, counting multiplities, inside C.

30. Show that the transformation $w = \frac{1}{z}$ tranforms circles and lines into circles and lines.

31. Find the bilinear transformation which maps the points 2, i, – 2 into the points 1, i, – 1 respectively.

32. Discuss the conformality of $f(z) = z^2$ at $z = 1 + i$. What is the angle of rotation and scale factor at $z = 1 + i$?

33. Show that every harmonic function defined on a simply connected domain has always a harmonic conjugate in that region.

Answer **any one** essay question out of 2. **(1×10=10)**

34. Let f be analytic at a point z_0. Prove that it has a zero of order m at z_0 if and only if there is a function g, which is analytic and non-zero at z_0, such that $f(z) = (z - z_0)^m g(z)$.

35. Using contour integration, prove that $\int_0^\infty \frac{\sin x}{x} dx = \frac{\pi}{2}$.
