



K17U 1389

Reg. No. :

Name :

**VI Semester B.Sc. Hon's (Mathematics) Degree (Regular/Supple./Improv.)
Examination, May 2017
BHM 604 : COMPLEX ANALYSIS – II
(2013 Admission)**

Time : 3 Hours

Max. Marks : 80

Answer **all** the **ten** questions :

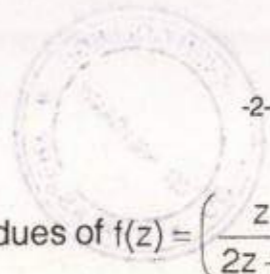
(10x1=10)

1. What are the isolated singular points of $f(z) = \frac{1}{\sin(\pi/z)}$?
2. Define residue of a complex function at infinity.
3. State Cauchy's Residue theorem.
4. State Jordan's Lemma.
5. What do you mean winding number ?
6. State Rouché's theorem.
7. Find the fixed points of the transformations $w = \frac{z-1}{z+1}$
8. Define a bilinear transformation.
9. What do you mean by critical points of a transformation ?
10. What do you mean by harmonic conjugate of a function ?

Answer **any 10** short answer questions out of 14 :

(10x3=30)

11. Find the residue of $f(z) = z \cos\left(\frac{1}{z}\right)$ at $z = 0$.
12. Find the poles and residues of $\cot z$.



13. Find the poles and residues of $f(z) = \left(\frac{z}{2z+1}\right)^3$.
14. Find the residue of $f(z) = \frac{\tanh z}{z^2}$ at $z = \frac{\pi i}{2}$.
15. What do you mean by zeroes of a complex function? Give an example.
16. If C denote the unit circle $|z| = 1$, described in the positive sense, determine the winding number of $f(z) = \frac{1}{z^2}$.
17. Determine the number of zeroes counting multiplicities of the polynomial $z^6 - 5z^4 + z^3 - 2z$ inside the circle $|z| = 1$.
18. What is the image of the half plane $x > 0$ under the transformation $w = iz + i$?
19. Write a note on the transformation $w = \frac{1}{z}$.
20. Find the image of the line $x = 1$ under the map $w = \frac{1}{z}$.
21. Show that a bilinear transformations maps circles and lines into circles and lines.
22. What do you mean by an isogonal mapping? Give an example.
23. What do you mean by local inverse of a conformal mapping at a point?
24. Explain angle of rotation and scale factor of a conformal mapping.

Answer **any 6** short answer questions out of 9: (6×5=30)

25. Evaluate $\int_C \exp\left(\frac{1}{z^2}\right) dz$ where C is the positively oriented unit circle $|z| = 1$.

26. State and prove Cauchy's Residue theorem.

27. Suppose that z_0 is an essential singularity of a function f and let w_0 be any complex number. Then prove that for any positive number ϵ , the inequality $|f(z) - w_0| < \epsilon$ is satisfied at some point z in each deleted neighborhood $0 < |z - z_0| < \delta$ of z_0 .
28. Convert the integral $\int_0^{2\pi} \frac{d\theta}{1 + a \sin \theta}$, ($-1 < a < 1$) into a complex integral and hence evaluate it.
29. Using Rouché's theorem, deduce the fundamental theorem of algebra.
30. Show that the transformation $w = \frac{1}{z}$ transforms lines into circles and lines.
31. Find the bilinear transformation which maps the points $1, 0, -1$ into the points $i, \infty, 1$ respectively.
32. Find the angle of rotation and scale factor at the point $1 + i$ when $w = z^2$.
33. Find the harmonic conjugate of $u(x, y) = x^3 - 3xy^2$. Also write the resulting analytic function in terms of the complex variable z .

Answer **any one** essay questions out of 2:

(1×10=10)

34. If a function f is analytic everywhere in the finite plane except for a finite number of singular points interior to a positively oriented simple closed contour C , then prove that

$$\int_C f(z) dz = 2\pi i \operatorname{Res}_{z=0} \left[\frac{1}{z^2} f\left(\frac{1}{z}\right) \right].$$

35. Evaluate $\int_{-\infty}^{\infty} \frac{x \sin x}{x^2 + 2x + 2} dx$.