K17U 1389

Reg. No. : ....

Name : ....

VI Semester B.Sc. Hon's (Mathematics) Degree (Regular/Supple./Improv.) Examination, May 2017
BHM 604 : COMPLEX ANALYSIS – II
(2013 Admission)

Time : 3 Hours Max. Marks : 80

Answer **all** the **ten** questions : **(10×1=10)**

1. What are the isolated singular points of $f(z) = \frac{1}{\sin(\pi / z)}$?
2. Define residue of a complex function at infinity.
3. State Cauchy's Residue theorem.
4. State Jordan's Lemma.
5. What do you mean winding number ?
6. State Rouche's theorem.
7. Find the fixed points of the transformations $w = \frac{z-1}{z+1}$
8. Define a bilinear transformation.
9. What do you mean by critical points of a transformation ?
10. What do you mean by harmonic conjugate of a function ?

Answer **any 10** short answer questions out of 14 : **(10×3 =30)**

11. Find the residue of $f(z) = z \cos\left(\frac{1}{z}\right)$ at $z = 0$.
12. Find the poles and residues of cotz.

P.T.O.

13. Find the poles and residues of $f(z) = \left(\frac{z}{2z+1}\right)^3$

14. Find the residue of $f(z) = \frac{\tanh z}{z^2}$ at $z = \frac{\pi i}{2}$

15. What do you mean by zeroes of a complex function ? Give an example.

16. If C denote the unit circle |z| = 1, described in the positive sense, determine the winding number of $f(z) = \frac{1}{z^2}$.

17. Determine the number of zeroes counting multiplicities of the polynomial $z^6 - 5z^4 + z^3 - 2z$ inside the circle |z| = 1.

18. What is the image of the half plane x > 0 under the transformation $w = iz + i$?

19. Write a note on the transformation $w = \frac{1}{z}$.

20. Find the image of the line x = 1 under the map $w = \frac{1}{z}$

21. Show that a bilinear transformations maps circles and lines into circles and lines.

22. What do you mean by an isogonal mapping ? Give an example.

23. What do you mean by local inverse of a conformal mapping at a point ?

24. Explain angle of rotation and scale factor of a conformal mapping.

Answer **any 6** short answer questions out of 9 : **(6×5 =30)**

25. Evaluate $\int_C \exp\left(\frac{1}{z^2}\right) dz$ where C is the positively oriented unit circle |z| = 1.

26. State and prove Cauchy's Residue theorem.

27. Suppose that z_0 is an essential singularity of a function f and let w_0 be any complex number. Then prove that for any positive number ϵ, the inequality $|f(z) - w_0| < \epsilon$ is satisfied at some point z in each deleted neighborhood $0 < |z - z_0| < \delta$ of z_0.

28. Convert the integral $\int_0^{2\pi} \frac{d\theta}{1 + a\sin\theta}, (-1 < a < 1)$ into a complex integral and hence evaluate it.

29. Using Rouche's theorem, deduce the fundamental theorem of algebra.

30. Show that the transformation $w = \frac{1}{z}$ transforms lines into circles and lines.

31. Find the bilinear transformation which maps the points 1, 0, –1 into the points i, ∞, 1 respectively.

32. Find the angle of rotation and scale factor at the point 1 + i when $w = z^2$.

33. Find the harmonic conjugate of $u(x, y) = x^3 - 3xy^2$. Also write the resulting analytic function in terms of the complex variable z.

Answer **any one** essay questions out of 2 : **(1×10 =10)**

34. If a function f is analytic everywhere in the finite plane except for a finite number of singular points interior to a positively oriented simple closed contour C, then prove that

$$\int_C f(z)\, dz = 2\pi i \ \text{Res}_{z=0}\left[\frac{1}{z^2} f\left(\frac{1}{z}\right)\right].$$

35. Evaluate $\int_{-\infty}^{\infty} \frac{x \sin x}{x^2 + 2x + 2} dx$.
