Reg. No. : ....

Name : ....

VI Semester B.Sc. Hon's (Mathematics) Degree (Regular) Examination, May 2016
BHM – 604 : COMPLEX ANALYSIS – II

Time : 3 Hours

Max. Marks : 80

Answer **all** the **ten** questions :

(10×1=10)

1. What do you mean by an isolated singular point ?

2. Define the term 'residue' of a complex function at a singular point.

3. What do you mean by a pole ?

4. State Jordan's Lemma.

5. What do you mean by meromorphic functions ?

6. State argument principle.

7. What do you mean by fixed points of a transformation ?

8. Define a linear fractional transformation.

9. Find the critical points of the transformation $w = z^2$.

10. What do you mean by harmonic conjugate of a function ?

Answer **any 10** short answer questions out of **14** :

(10×3=30)

11. Find the residue of $f(z) = \dfrac{1}{z + z^2}$ at $z = 0$.

12. What are the different types of isolated singular points ? Give examples in each case.

13. Find the residue of $f(z) = \dfrac{z}{z^4 + 4}$ at the isolated singular point $z_0 = 1 + i$.

14. Evaluate $\displaystyle\int_C \tan z \, dz$ where C is positively oriented circle $|z| = 2$.

15. Find the zeroes and their orders of $f(z) = z(e^z - 1)$.

16. If C denote the unit circle $|z| = 1$, described in the positive sense, determine the winding number of $f(z) = z^2$.

17. Determine the number of roots of the equation $z^7 - 4z^3 + z - 1 = 0$ inside the circle $|z| = 1$.

18. What is the image of the infinite strip $0 < x < 1$ under the transformation $w = iz$?

19. Describe the transformation $w = \dfrac{1}{z}$.

20. Find the image of the line $y = c$ under the map $w = \dfrac{1}{z}$.

21. Express the bilinear transformation as a composition of linear transformations and inversions.

22. What do you mean by a conformal mapping ? Give an example.

23. Find the points where $w = \sin z$ is not conformal.

24. Find the local inverse of the transformation $w = e^z$ at the point $2\pi i$.

Answer **any 6** short answer questions out of **9** :   **(6×5=30)**

25. Evaluate $\displaystyle\int_C z^2 \sin\left(\dfrac{1}{z}\right) dz$ where C is the positively oriented unit circle $|z| = 1$.

26. State and prove Cauchy's Residue theorem.

27. If z_0 is a pole of a function f, prove that $\lim_{z \to z_0} f(z) = \infty$.

28. Using Residue theorem evaluate the integral $\displaystyle\int_0^{2\pi} \dfrac{d\theta}{(5 - 3\cos\theta)^2}$.

29. State and prove Rouche's theorem.

30. Show that the transformation $w = \dfrac{1}{z}$ transforms circles into circles and lines.

31. Find the bilinear transformation which maps the points $-1, 0, 1$ into the points $-i, 1, i$ respectively.

32. What do you mean by angle of rotation and scale factor ? Also find the angle of rotation and scale factor at the point $1 + i$ when $w = z^2$.

33. Find the harmonic conjugate of $u(x, y) = xy$. Also write the resulting analytic function in terms of the complex variable z.

Answer **any one** essay questions out of **2** :   **(1×10=10)**

34. Let f be analytic at a point z_0. Prove that it has a zero of order m at z_0 if and only if there is a function g, which is analytic and non-zero at z_0, such that $f(z) = (z - z_0)^m g(z)$.

35. Evaluate $\displaystyle\int_0^\infty \dfrac{x^2}{x^6 + 1} dx$.