K19U 0778

Reg. No. : ....

Name : ....

VI Semester B.Sc. Hon's (Mathematics) Degree (Supplementary) Examination, April 2019
(2013-15 Admissions)
BHM 604 : COMPLEX ANALYSIS – II

Time : 3 Hours　　　　Max. Marks : 80

Answer **all** the **ten** questions. **(10×1=10)**

1. Define isolated singular points of the function.
2. Find the fixed point of the transformation $\omega = \frac{z-1}{z+1}$.
3. Find the zero and its order of the function $f(z) = z(e^z - 1)$.
4. State Riemann's Theorem.
5. Find Cauchy principal value of the integral $\int_{-\infty}^{\infty} x\,dx$.
6. Define bilinear transformation.
7. Find the image of the circle not passing through origin under the transformation $f(z) = \frac{1}{z}$.
8. Define translation mapping.
9. Define harmonic conjugate of a function.
10. Find the Jacobian of the transformation $f(z) = x^2 + iy^2$.

Answer **any 10** short answer questions out of 14. **(10×3=30)**

11. Find the residue at z = 0 of the function $f(z) = \frac{z - \sin z}{z}$.
12. Using residue show that $\int_c \exp\left(\frac{1}{z^2}\right) dz = 0$, where c is the positively oriented circle $|z| = 1$.

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13. Find the poles and residues of the function $f(z) = \frac{1}{z^2(1+z)}$.

14. Determine the value of $\Delta_c \arg f(z)$ when $f(z) = \frac{z^2+2}{z}$.

15. Determine the number of zeros counting multiplicities of the polynomial $2z^5 - 6z^2 + z + 1$ in the annulus $1 < |z| < 2$.

16. Evaluate the integral $\int_c \frac{dz}{z^3(z+4)}$, where c denote the positively oriented circle $|z| = 2$.

17. Find the image of the half plane $x < c_1$, $(c_1 < 0)$ under the transformation $\omega = \frac{1}{z}$.

18. Write a short note on linear fractional transformation.

19. Prove that if z_0 is a pole of a function f, then $\lim_{z \to z_0} f(z) = \infty$.

20. Show that the transformation $\omega = \frac{1}{z}$ transforms lines into circles.

21. Find the pole and residues of the function $f(z) = \frac{\tanh z}{z^2}$.

22. Find the harmonic conjugate of the function $v = -\frac{1}{z}x^2 + \frac{1}{z}y^2$.

23. Find the angle of rotation produced by the transformation $\omega = \frac{1}{z}$ at the point $z_0 = i$.

24. What do you mean by local inverse of a conformal mapping at a point ?

Answer **any 6** short answer questions out of 9. **(6×5=30)**

25. Evaluate the integral of the function $f(z) = \frac{z+1}{z^2 - 2z}$ around the circle $|z| = 3$ in the positive sense by using the Cauchy's Residue Theorem.

26. Let a function f be analytic at a point z_0. Suppose f has a zero of order n at z_0. Then prove that there exists a function g which is analytic and non zero at z_0 such that $f(z) = (z - z_0)^n g(z)$.

27. Let p and q are analytic at a point z_0. Prove that if $p(z_0) \neq 0$ and $q^1(z_0) \neq 0$, then prove that z_0 is a simple pole of the quotient $\frac{p(z)}{q(z)}$ and $\mathrm{Res}_{z \to z_0} \frac{p(z)}{q(z)} = \frac{p(z_0)}{q^1(z_0)}$.

28. Use residue to evaluate the integral $\int_{-\pi}^{\pi} \frac{d\theta}{1+\sin^2\theta}$.

29. Find Cauchy Principal value of the integral $\int_{-\infty}^{\infty} \frac{(x+1)\cos x \, dx}{x^2 + 4x + 5}$.

30. Find the linear transformation which maps the point (– i, 0 i) into the point (– i, i, 1).

31. State and prove Cauchy's Residue theorem.

32. Find the local inverse of the transformation $\omega = z^2$ at the point $z_0 = -i$.

33. Suppose u (x, y) is any given harmonic function defined on a simply connected domain D. Show that u (x, y) always has a harmonic conjugate v (x, y) in D, deriving an expression for v (x, y).

Answer **any one** essay question out of 2. **(1×10=10)**

34. Prove that : An isolated singular point z_0 of a function f is a pole of order n if and only if f (x) can be written in the form $f(z) = \frac{Q(z)}{(z - z_0)^n}$, where Q (z) is analytic and non zero at z_0. Moreover, $\mathrm{Res}_{z \to z_0} f(z) = Q(z_0)$, if n = 1 and $\mathrm{Res}_{z \to z_0} f(z) = \frac{Q^{n-1}(z_0)}{(n-1)!}$, if $n \geq 2$.

35. a) State and prove Rouche's Theorem.

    b) Determine the number of roots of the equation $z^7 - 4z^3 + z - 1 = 0$ inside the circle $|z| = 1$.

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