31. If heat is generated at a constant rate throughout a bar of length $L = \pi$ with initial temperature $f(x)$ and the ends at $x = 0$ and π are kept at temperature 0, the heat equation is $u_t = c^2 u_{xx} + H$ with constant $H > 0$. Solve the problem.

32. Find the potential in the rectangle $0 \le x \le 20$, $0 \le y \le 40$ where upper side is kept at potential 220V and whose other sides are grounded.

33. Find the general solution of $(1+x^2)y'' + 2xy' - 2y = 0$ in terms of power series in x. Can you express the solution by means of elementary functions. **(6×5=30)**

SECTION – D

Answer **any 1** questions out of 2 questions. **Each** question carries **10** marks.

34. What is least square approximation of Legendre polynomials ?

35. A rod of length l is heated so that its ends A and B are at zero temperature. If initially its temperature is given by $u = \frac{cx(l-x)}{l^2}$, find the temperature at time t.

(1×10=10)

Reg. No. : ………………………………

Name : …………………………………

V Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, November 2017

BHM503 : SPECIAL FUNCTIONS AND PARTIAL DIFFERENTIAL EQUATIONS

(2013 – 15 Admissions)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. What is the general form of homogeneous second order linear differential equation ?
2. What is the general form of Legendre's equation ?
3. What is Γp ?
4. Prove that $\frac{d}{dx}\left[x^p J_p(x)\right] = x^p J_{p-1}(x)$.
5. Define a periodic function.
6. What is the Euler formula for a_0 ?
7. Define an even function.
8. What is the general form of one dimensional wave equation ?
9. What is the Fourier series solution of heat equation ?
10. State Bessel's expansion theorem. **(10×1=10)**

P.T.O.

SECTION – B.

Answer **any 10** questions out of 14 questions. **Each** question carries **3** marks.

11. Use the ratio test to verify that R = 0, ∞ and 1 for the series

i) $\sum_0^\infty n!x^n$ ii) $\sum_0^\infty \frac{x^n}{n!}$ iii) $\sum_0^\infty x^n$.

12. Determine the nature of the point x = 0 for the differential equation $x^2y'' + \sin xy = 0$.

13. Verify that $(1 + x)p = F(-p, b, b, -x)$.

14. Find the first two terms of the Legendre series of

$f(x) = 0$ if $-1 \le x \le 0$

$= x$ if $0 \le x \le 1$

15. Prove that $\Gamma p = (p-1)\,\Gamma(p-1)$.

16. Prove that $\cos x = J_0(x) - 2J_2(x) + 2J_4(x) - .....$

17. Find the Fourier series of the function

$f(x) = 0$ if $-2 < x, -1$

$= k$ if $-1 < x < 1$

$= 0$ if $1 < x < 2$, where $p = 2L = 4$, $L = 2$.

18. Find the temperature u (x, t) in a laterally insulated copper bar 80 cm long if the initial temperature is $100\sin\left(\pi x/80\right)^\circ$ C and the ends are kept at 0°c. How long it will take for the maximum temperature of the bar to drop to 50°C ? Physical data for copper density 892 gm/cm^3, specific heat 0.092cal/(gm°c), thermal conductivity 0.95cal/(cmsec°c).

19. Find the power series solution of $y' = y$.

20. What is meant by regular singular point ?

21. State the orthogonal property of Legendre polynomial.

22. Obtain the solution of the heat equation $\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$ by the method of separation of variables.

23. Expand $f(x) = x^3$ in $-\pi < x < \pi$ in a Fourier series.

24. Find half range cosine series for $f(x) = x^2$ in $0 \le x \le \pi$. **(10×3=30)**

SECTION – C

Answer **any 6** questions out of 9 questions. **Each** question carries **5** marks.

25. Find the nature of the point at infinity for the differential equation $x^2y'' + 4xy' + 2y = 0$.

26. Derive the Rodrigue's formula for Legendre polynomials.

27. With the usual notation prove that

i) $J_0'(x) = -J_1(x)$

ii) $\frac{d}{dx}[xJ_1(x)] = xJ_0(x)$.

28. Show that in the range, $0 \le x \le \pi$, $x(\pi - x) = \frac{\pi^2}{6} - \left(\frac{\cos 2x}{1^2} + \frac{\cos 4x}{2^2} + \frac{\cos 6x}{3^2} + ...\right)$.

29. Find the type, transform to normal form and solve $u_{xx} - 2u_{xy} + u_{yy} = 0$.

30. Find the temperature in a laterally insulated bar of length L whose ends are insulated, assuming that the initial temperature is

$f(x) = x$ if $0 < x < \frac{L}{2}$

$= L - x$ if $\frac{L}{2} < x < L$.