Reg. No. : ....

Name : ....

K16U 2588

V Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, November 2016
BHM503 : SPECIAL FUNCTIONS & PARTIAL DIFFERENTIAL EQUATIONS

Time : 3 Hours   Max. Marks : 80

Answer **all** the **ten** questions.   **(Marks : 10×1=10)**

1. What do you mean by the radius of convergence of a power series ?
2. Define the term 'singular point' of a second order linear differential equation.
3. What is the nature of the point $x = 0$ for the differential equation $y'' + (\sin x)\, y = 0$?
4. Write the generating function for the Legendre polynomials.
5. Define Bessel function of order 1.
6. What is the period of $f(x) = \sin x$?
7. Define Fourier sine series expansion of a function in the interval (0, L).
8. What do you mean by an even function ?
9. Write two dimensional Laplace's equation.
10. What do you mean by a mixed problem ?

Answer **any 10** short answer questions out of **14**.   **(Marks : 10× 3=30)**

11. Find the power series solution of $y' = 2xy$.
12. Determine the nature of the point $x = \infty$ for $y'' + \frac{4}{x}y' + \frac{2}{x^2}y = 0$.
13. Find the indicial equation and its roots of $4x^2y'' + (2x^4 - 5x)y' + (3x^2 + 2)y = 0$.

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14. Prove that $P_{2n+1}(0) = 0$ and $P_{2n}(0) = (-1)^n \frac{1.3...(2n-1)}{2^n.n!}$.

15. Express $J_3(x)$ in terms of $J_0(x)$ and $J_1(x)$.

16. Prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x$.

17. Prove that $\Gamma(n) = n!$ if n is an integer.

18. Explain the convergence of Fourier series expansion of functions.

19. Obtain the Fourier coefficients a_0 and a_n in the expansion of

$$f(x) = \begin{cases} -k & \text{if } -\pi < x < 0 \\ k & \text{if } 0 < x < \pi \end{cases}.$$

20. Express $f(x) = x^2$ as a half range cosine series in $0 < x < \pi$.

21. Solve the partial differential equation $u_{xy} = -u_x$.

22. What are the assumptions involved in the derivation of one dimensional wave equations ?

23. Write the possible boundary conditions and initial conditions of one dimensional heat equation if ends of the bar are insulated.

24. What is the superposition principle in the solution of partial differential equations ?

Answer **any 6** short answer questions out of **9**. **(Marks : 6× 5=30)**

25. Find the power series solution of $(1 - x^2) y'' - 2xy' + p(p + 1) y = 0$.

26. Find the power series solution of Gauss hypergeometric equation

$$x(1 - x) y'' + [c - (a + b + 1) x] y' - aby = 0.$$

27. Derive Rodrigue's formula.

28. Prove that $2J'_p(x) = J_{p-1}(x) - J_{p+1}(x)$.

29. Obtain the Fourier series for the function $f(x) = x + \pi$ in the interval $(-\pi, \pi)$.

30. Find the Fourier series expansion of e^{-x} in the interval $(-l, l)$.

31. Using the method of separation of variables, solve $u_{xy} - u = 0$.

32. Derive one dimensional wave equation.

33. Obtain all the possible solution of Laplace equation in two variables by the method of separation of variables.

Answer **any one** essay question out of **2**. **(Marks : 1× 10=10)**

34. State and prove orthogonality property of Bessel's function.

35. Obtain the Fourier series for the function $f(x) = x - x^2$ in the interval $(-\pi, \pi)$.

Deduce that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - ... = \frac{\pi^2}{12}$.
