K15U 0347

Reg. No. : .....

Name : .....

V Semester B.Sc. (Hon's) (Mathematics) Degree (Regular) Examination, November 2015

BHM 503 : SPECIAL FUNCTIONS AND PARTIAL DIFFERENTIAL EQUATIONS

Time : 3 Hours Max. Marks : 80

Answer **all** the **ten** questions. **(10×1=10)**

1. Write the Taylor's series formula for f(x).
2. What do you mean by regular singular point of a second order linear differential equation ?
3. Locate and classify the singular point on the x-axis of

   $x^3 (x-1) y'' - 2 (x-1) y' + 3xy = 0$.
4. What do you mean by Bessel's equation and Bessel's function ?
5. Define Gamma function.
6. What do you mean by a periodic function ?
7. Define Fourier series.
8. Give an example for an odd function.
9. Write one dimensional wave equation.
10. What do you mean by a Dirichlet problem ?

Answer **any 10** short answer questions out of **14**. **(10×3=30)**

11. Find the power series solution of $y' = y$.
12. Determine the nature of the point $x = \infty$ for $(1 - x^2) y'' - 2xy' + p (p+1) y = 0$.
13. Find the indicial equation and its roots of $x^3 y'' + (\cos 2x - 1) y' + 2xy = 0$.
14. Write the first three Legendre polynomials.
15. Show that $\frac{d}{dx} J_0 (x) = -J_1 (x)$.

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16. Express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$.

17. Prove that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$.

18. Explain the Fourier sine series and cosine series expansion of functions in (0, L).

19. Obtain the Fourier coefficients a_0 and a_n in the expansion of $f(x) = x \cos x$ in the interval $(-\pi, \pi)$.

20. Express $f(x) = e^x$ as a half range sine series in $0 < x < 1$.

21. Solve the partial differential equation $u_{xx} - u = 0$.

22. What are the assumptions involved in the derivation of one dimensional wave equation ?

23. Write the possible boundary conditions and initial conditions of one dimensional heat equation.

24. What are elliptic, parabolic and hyperbolic equations ? Give examples.

Answer **any 6** short answer questions out of **9**. **(6×5=30)**

25. Find the power series solution of $y'' + y = 0$.

26. Find the power series solution of $2x^2y'' + x(2x + 1)\, y' - y = 0$ using the method of Frobenius.

27. Derive Rodrigues' formula.

28. Prove that $\frac{d}{dx}\left[x^p J_p(x)\right] = x^p J_{p-1}(x)$.

29. Obtain the Fourier series for the function $f(x) = x + x^2$, $-\pi < x < \pi$.

30. Find the Fourier series expansion of $f(x) = x^2$ in the interval $(-1, 1)$.

31. Using the method of separation of variables, solve $u_{xx} - u_{yy} = 0$.

32. Obtain D'Alembert's solution for one dimensional wave equation.

33. Explain the method of separation of variables to solve one dimensional heat equation.

Answer **any one** essay questions out of **2**. **(1×10=10)**

34. State and prove orthogonality property of Legendre polynomials.

35. Obtain the Fourier series for the function $f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2-x), & 1 \le x \le 2 \end{cases}$

Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots\ldots\ldots = \frac{\pi^2}{8}$.