



Reg. No. : .....

Name : .....

V Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, November 2020 (2016 Admission Onwards) BHM 501 : SPECIAL FUNCTIONS

Time : 3 Hours

Max. Marks : 60

SECTION - A

Answer any 4 questions out of 5 questions. Each question carries 1 mark.

- 1. Define a power series.
2. Define a singular point of a second order linear homogeneous differential equation.
3. Give an example of a differential equation with a regular singular point at x = -1.
4. Write down the nth Legendre Polynomial Pn(x).
5. Define the gamma function. (4x1=4)

SECTION - B

Answer any 6 questions out of 9 questions. Each question carries 2 marks.

- 6. Define an ordinary point of a second order linear homogeneous differential equation and give an example of a differential equation with an ordinary point.
7. For the differential equation (3x + 1)xy'' - (x + 1)y' + 2y = 0, locate and classify its singular points on the x-axis.
8. Determine the nature of the point x = 0 for the differential equation y'' + (sinx)y = 0.
9. Write down the Gauss's hypergeometric differential equation and its solution F(a, b, c, x).
10. Verify that e^x = lim\_{b -> infinity} F(a, b, a, x/b) by examining the series expansions of the functions on the left sides.

11. State the orthogonality property of Legendre polynomials.
12. Write short notes on Legendre series.
13. Prove that  $\frac{d}{dx} [x^p J_p(x)] = x^p J_{p-1}(x)$ .
14. State the quotient test for an improper integral of second kind. **(6×2=12)**

## SECTION - C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. Find a power series solution of the form  $\sum_{n=0}^{\infty} a_n x^n$  to solve the differential equation  $xy' = y$  and verify your conclusion by solving the equation directly.
16. Check the nature of the point  $x = 0$  for the differential equation  $y'' + y' - xy = 0$  and find a power series solution for it which satisfy the conditions  $y_2(0) = 0$ ,  $y_2'(0) = 1$ .
17. Find the indicial equation and its roots for the differential equation  $2x^2 y'' + x(2x + 1)y' - y = 0$ .
18. Write down the hypergeometric series  $F(a, b, c, x)$  and hence verify that the following by examining the series expansions of the functions on the left sides :
- i)  $(1 + x)^p = F(-p, b, b, -x)$ ;
- ii)  $\sin^{-1}x = xF\left(\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, x^2\right)$ .
19. Find the general solution of  $(2x^2 + 2x)y'' + (1 + 5x)y' + y = 0$  near the singular point  $x = 0$ , in terms of Gauss's hypergeometric series.
20. Determine the nature of the point  $x = \infty$  for the differential equation  $x(1 - x)y'' + [c - (a + b + 1)x]y' - aby = 0$ .
21. Verify that  $(1 - 2xt + t^2)^{-1/2}$  is the generating function for Legendre polynomials.
22. Find the first three terms of the Legendre series of  $f(x) = e^x$ .
23. Prove that i)  $\frac{d}{dx} J_0(x) = -J_1(x)$  and ii)  $\frac{d}{dx} [xJ_1(x)] = xJ_0(x)$ .

24. Classify according to the type of improper integral :

i)  $\int_{-1}^1 \frac{1}{\sqrt[3]{x(x+1)}} dx$     ii)  $\int_0^{\pi} \frac{1 - \cos x}{x^2} dx$     iii)  $\int_{-\infty}^{\infty} \frac{x^2}{x^4 + x^2 + 1} dx$ .

25. Identify the type of improper integral  $\int_1^{\infty} \frac{1}{x\sqrt{(2x-1)}} dx$ . Convert this integral into an integral of the second kind and into a proper integral.

26. Prove that Beta function is symmetric and  $B(m, n) = 2 \int_0^{\pi/2} \cos^{2m-1}\theta \sin^{2n-1}\theta d\theta$ . **(8×4=32)**

## SECTION - D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. Find two independent Frobenius series solutions for the differential equation  $2xy'' + (3 - x)y' - y = 0$ .
28. Determine the nature of the point  $x = \infty$  for the Legendre's equation  $(1 - x^2)y'' - 2xy' + p(p + 1)y = 0$ , and find the exponents from the indicial equation.
29. State and prove the orthogonality property of Bessel functions.
30. Prove that  $B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$ ,  $m, n > 0$ . **(2×6=12)**