K21U 0221

Reg. No. : ....

Name : ....

V Semester B.Sc. Hon's (Mathematics) Degree (Reg./Supple./Improv.) Examination, November 2020
(2016 Admission Onwards)
BHM 501 : SPECIAL FUNCTIONS

Time : 3 Hours Max. Marks : 60

SECTION – A

Answer **any 4** questions out of 5 questions. **Each** question carries **1** mark.

1. Define a power series.
2. Define a singular point of a second order linear homogeneous differential equation.
3. Give an example of a differential equation with a regular singular point at $x = -1$.
4. Write down the n^{th} Legendre Polynomial $P_n(x)$.
5. Define the gamma function. **(4×1=4)**

SECTION – B

Answer **any 6** questions out of 9 questions. **Each** question carries **2** marks.

6. Define an ordinary point of a second order linear homogeneous differential equation and give an example of a differential equation with an ordinary point.
7. For the differential equation $(3x + 1)xy'' - (x+1)y' + 2y = 0$, locate and classify its singular points on the x-axis.
8. Determine the nature of the point $x = 0$ for the differential equation $y'' + (\sin x)\, y = 0$.
9. Write down the Gauss's hypergeometric differential equation and its solution $F(a, b, c, x)$.
10. Verify that $e^x = \lim_{b \to \infty} F\left(a, b, a, \frac{x}{b}\right)$ by examining the series expansions of the functions on the left sides.

P.T.O.

11. State the orthogonality property of Legendre polynomials.

12. Write short notes on Legendre series.

13. Prove that $\frac{d}{dx}\left[x^p J_p(x)\right] = x^p J_{p-1}(x)$.

14. State the quotient test for an improper integral of second kind. **(6×2=12)**

SECTION – C

Answer **any 8** questions out of 12 questions. **Each** question carries **4** marks.

15. Find a power series solution of the form $\sum_{n=0}^{\infty} a_n x^n$ to solve the differential equation $xy' = y$ and verify your conclusion by solving the equation directly.

16. Check the nature of the point $x = 0$ for the differential equation $y'' + y' - xy = 0$ and find a power series solution for it which satisfy the conditions $y_2(0) = 0$, $y_2'(0) = 1$.

17. Find the indicial equation and its roots for the differential equation $2x^2 y'' + x(2x + 1)y' - y = 0$.

18. Write down the hypergeometric series F(a, b, c, x) and hence verify that the following by examining the series expansions of the functions on the left sides :

    i) $(1 + x)^p = F(-p, b, b, -x)$;

    ii) $\sin^{-1}x = xF\left(\frac{1}{2}, \frac{1}{2}, \frac{3}{2}, x^2\right)$.

19. Find the general solution of $(2x^2 + 2x)y'' + (1 + 5x)y' + y = 0$ near the singular point $x = 0$, in terms of Gauss's hypergeometric series.

20. Determine the nature of the point $x = \infty$ for the differential equation $x(1 - x)y'' + [c - (a + b + 1)x]y' - aby = 0$.

21. Verify that $(1 - 2xt + t^2)^{-1/2}$ is the generating function for Legendre polynomials.

22. Find the first three terms of the Legendre series of $f(x) = e^x$.

23. Prove that i) $\frac{d}{dx}J_0(x) = -J_1(x)$ and ii) $\frac{d}{dx}\left[xJ_1(x)\right] = xJ_0(x)$.

24. Classify according to the type of improper integral :

    i) $\int_{-1}^{1} \frac{1}{\sqrt[3]{x(x+1)}}dx$ ii) $\int_{0}^{\pi} \frac{1-\cos x}{x^2}dx$ iii) $\int_{-\infty}^{\infty} \frac{x^2}{x^4 + x^2 + 1}dx$.

25. Identify the type of improper integral $\int_{1}^{\infty} \frac{1}{x\sqrt{(2x-1)}}dx$. Convert this integral into an integral of the second kind and into a proper integral.

26. Prove that Beta function is symmetric and $B(m, n) = 2\int_{0}^{\pi/2} \cos^{2m-1}\theta \sin^{2n-1}\theta \, d\theta$. **(8×4=32)**

SECTION – D

Answer **any 2** questions out of 4 questions. **Each** question carries **6** marks.

27. Find two independent Frobenius series solutions for the differential equation $2xy'' + (3 - x)y' - y = 0$.

28. Determine the nature of the point $x = \infty$ for the Legendre's equation $(1 - x^2)y'' - 2xy' + p(p + 1)y = 0$, and find the exponents from the indicial equation.

29. State and prove the orthogonality property of Bessel functions.

30. Prove that $B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$, m, n > 0. **(2×6=12)**

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