SECTION - D

Answer any **2** questions out of **4** questions . Each question carries **6** Marks.

(2×6=12)

27. Find the only Frobenius series solution of the differential equation.

$x^2y'' - 3xy' + (4x+4)y = 0$.

28. Determine the nature of the point $x = \infty$ for the differential equation.

$x(1-x)y'' + [c-(a+b+1)x]y' - aby = 0$ and find the exponents from the indicial equation.

29. State and prove the orthogonality property of Bessel functions.

30. Show that $\int_0^{\pi/2} \cos^{2m-1}\theta \sin^{2n-1}\theta \, d\theta = \frac{\Gamma(m)\Gamma(n)}{2\Gamma(m+n)}, m,n > 0$.

Reg. No. : ....

Name : ....

V Semester B.Sc. Hon's (Mathematics) Degree(Reg./Supple./Improv.) Examination, November-2019

(2016 Addmission. Onwards)

BHM 501 : SPECIAL FUNCTIONS

Time : 3 Hours Max. Marks : 60

SECTION - A

Answer any **4** questions out of **5** questions . Each question carries **1** Mark. **(4×1=4)**

1. Write down the formula for calculating the radius of convergence of a power series.
2. Define an ordinary point of a second order linear homogeneous differential equation.
3. Give an example of a differential equation with a regular singular point at $x = 1$.
4. Write down the Bessel function of the first kind of order *p*.
5. Define the gamma function.

SECTION - B

Answer any **6** questions out of **9** questions . Each question carries **2** Marks. **(6×2=12)**

6. Define a singular point of a second order linear homogeneous differential equation and give an example of a differential equation with a singular point.
7. For the differential equation $x^2y'' + (2-x)y' = 0$, locate and classify its singular points on the x-axis.

8. Determine the nature of the point $x=0$ for the differential equation $xy''+(\sin x)y=0$.

9. Show that $F'(a,b,c,x)=\frac{ab}{c}F(a+1,\,b+1,\,c+1,\,x)$.

10. Verify that $\sin^{-1}x=xF\left(\frac{1}{2},\frac{1}{2},\frac{3}{2},x^2\right)$ by examining the series expansions of the functions on the left sides.

11. State the orthogonality property of Legendre polynomials.

12. Prove that $\frac{d}{dx}J_0(x)=-J_1(x)$.

13. Write short notes on Bessel series.

14. State the quotient test for an improper integral of first kind.

SECTION - C

Answer any **8** questions out of **12** questions . Exach question carries **4** Marks. **(8×4=32)**

15. Find a power series solution of the form $\sum_{n=0}^{\infty} a_n x^n$ to solve the differential equation $y'=2xy$ and verify your conclusion by solving the equation directly.

16. Check the nature of the point $x=0$ for the differential equation $y''+y=0$ and find a power series solution for it.

17. Find the indicial equation and its roots for the differential equation.

$$4x^2y''+(2x^4-5x)y'+(3x^2+2)y=0$$

18. Write down the hypergeometric series $F(a,b,c,x)$ and hence verify that the following by examing the series expansions of the functions on the left sides:

(i) $(1+x)^p=F(-p,b,b,-x);$

(ii) $e^x=\lim_{b\to\infty}F\left(a,b,a,\frac{x}{b}\right)$

19. Find the general solution of $x(1-x)y''+\left(\frac{3}{2}-2x\right)y'+2y=0$ near the singular point $x=0$, in terms of Gauss's hypergeometric series.

20. Determine the nature of the point $x=\infty$ for the Legendre's equation $(1-x^2)y''-2xy'+p(p+1)y=0$.

21. Derive the recursion formula $(n+1)p_{n+1}(x)=(2n+1)xp_n(x)-np_{n-1}(x)$.

22. Define the gamma function and prove that $\Gamma(n+1)=n!$.

23. Prove that (i) $\frac{d}{dx}\left[x^p j_p(x)\right]=x^p j_{p-1}(x)$ and (ii) $\frac{d}{dx}\left[x^{-p}j_p(x)\right]=-x^{-p}j_{p+1}(x)$.

24. Classify according to the type of improper integral:

(i) $\int_2^{10}\frac{x}{(x-2)^3}dx$

(ii) $\int_0^{\infty}\frac{1}{1+\tan x}dx$

(iii) $\int_{-\infty}^{\infty}\frac{x^2}{x^4+x^2+1}dx$

25. Identify the type of improper integral $\int_1^2\frac{1}{\sqrt{x(2-x)}}dx$. Convert this integral into an integral of the first kind and in to a proper integral.

26. Prove that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$.